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THE
COMMON SENSE OF POLITICAL ECONOMY
AND
SELECTED PAPERS AND REVIEWS
ON ECONOMIC THEORY
VOL. II

**THE COMMON SENSE
OF
POLITICAL ECONOMY
AND
SELECTED PAPERS AND REVIEWS
ON ECONOMIC THEORY**

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VOL. II



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BOOK II

EXCURSIVE AND CRITICAL

Cum rerum natura nusquam magis quam in minimis tota sit.

PLINY THE ELDER.

Nowhere is the nature of things more intimately revealed than in the calculus of infinitesimals.

CHAPTER I

MARGINS AND THEIR DIAGRAMMATIC REPRESENTATION

SUMMARY.--*This chapter is devoted to a fuller examination of the principle of declining marginal significances. It is always the provocatives, opportunities, or supports of desired experiences or vents of impulse, and never those experiences themselves, that this law illustrates; but within that area it seems to be universal. It may appear, at first sight, that the claims of duty, of faith, or of humanity are not (or at least should not be) subject to any declining urgency as they are more fully met; and also that some satisfactions are habitually indulged in down to the point of satiety, whereas, according to our theory, the last and least significant increments of the things that minister to them should be less valued than increments of other things that would minister to still unsatisfied wants. But a careful examination will shew that these objections either rest on some misapprehension or are due to the fact that, under any given set of conditions, there is always a "minimum sensible" below which conscious estimates cannot be carried. Another set of difficulties arises from a confusion between the positive and negative sign of increments of satisfaction and a positive or negative state of satisfaction. The attempt to dispel this confusion, in connection with the diagrammatic method, leads us to an examination of the reactions of various kinds of indulgence upon the organism itself and its future capacities for enjoyment. This again leads to the discovery of interesting relations between a hedonistic calculus and current moral judgments. Our method, however, does not imply a hedonistic theory*

of conduct. The chapter closes with some notes on the dangers and limitations of the diagrammatic method it has introduced.

The whole structure raised in the First Book of this treatise rests upon the principle of declining marginal significance as supplies increase; and though we have established and illustrated it with sufficient firmness and accuracy for the immediate purposes of that Book, yet a number of problems to which no precise answers have been given may well present themselves to the reflective reader; and the extreme importance of the principle itself makes it desirable that it should be investigated and tested, not only in its immediate applications to economic problems, but in its fuller scope. Any misgiving as to its general validity might throw a taint of suspicion on its special applications. Moreover, we shall find that the closer investigation upon which we are now to enter will throw much light upon the connection between the narrower problems of Economics and the broader problems of Sociology; or perhaps we might say, between commercial Economics and the true Political Economy, in the sense of the economy of the *polis*, or regulation of the resources of the community.

Let us begin by noting that in speaking of declining significance we are never dealing with the ultimately desired experiences themselves, but always with something that we value as likely to produce such experiences. Thus, we spoke of concerts which a man wishes to attend because he thinks he will derive enjoyment from them; and we saw that, other things being equal, he would value a fifth concert per week less than a fourth. We did not say that a fifth "unit of enjoyment of music" would be less valuable to him than a fourth, for our only conception of a unit of enjoyment must be a quantity of enjoyment which equals some standard amount; so that each unit, being equal to the standard, would be equal to every other unit, and to say that the fifth unit was of less value than the fourth would be to say that two amounts were equal to the same but not equal to each other. Indeed it would obviously be nonsense to say that equally desired experiences have a declining sig-

The law of
declining
marginal
significance.

Distinction
between
experiences
and the things
that generate
them.

nificance, for if their significance declines they are not equally desired. In the same way, if we declare that opportunities of study have a declining value to a man, we may mean that if he has twelve hours a day clear for study he will attach less value to a thirteenth hour than he would to a fifth hour if he had only four; but we can hardly mean that successive acquisitions of a unit of information have a declining value, for we can hardly define a unit of information; and we cannot mean that successive increments of the pleasure or advantage he derives from the results of his study have declining value, for our only conception of equal increments of satisfaction must be increments that have the same value. And so throughout. So we are never speaking, in this connection, of units of experience, which (if we can form any conception of them at all) must be regarded as equal, but of units objectively measurable, roughly or accurately—whether by time, space, weight, number, or otherwise,—which are valued for the sake of the states of consciousness they are expected to produce or the vent they afford to impulses.

What we assert, then, is that after a certain point successive increments of external stimulants, or opportunities, produce successively declining increments of the desired internal experiences. And this principle applies not only to things provocative of delight to the senses, but to means of artistic and literary enjoyment, and even to opportunities for securing the satisfactions, or obeying the impulses, of friendship or affection. But it is sometimes asked, "Is not the case different when questions of duty are concerned? Does not duty always remain paramount, however much of your powers and resources you have already devoted to its demands? And are not the claims of compassion always superior to those of selfishness, however much you may have indulged the former and starved the latter? Is it possible for a well-regulated mind to bring about a marginal coincidence of value between the means of satisfying desires which are on essentially different ethical levels? Can such qualitative distinctions be reduced to questions of quantity?" That they are so reduced, it will be admitted, is a fact (whether lamentable or not), and in dealing with ordinary humanity we might be safe enough in assuming that such a reduction

would take place; but when we find that the martyr who has borne the rack is ready to be burnt to death sooner than depart a hair's breadth from the formula of his confession, we seem to have reached a region to which this law of diminishing significance does not apply. However much the martyr has given to his faith and however little he has kept for his comfort, it would appear that the escape from no quantity of physical anguish, however great, will weigh against any concession in the matter of faith, however small.

Such questions may seem to take us very far from our proper subject, and so indeed they do, and it is for this reason that they have been excluded from consideration at an earlier period. But I have maintained from first to last that the laws of Economics are the laws of life, and consequently if a law declares itself to be paramount on the economic field, it proclaims itself by implication as a general law of life and conduct. It may therefore be legitimately challenged on any field, and if it cannot hold its own everywhere it must at least lie under suspicion in its economic applications. In any case, a closer inspection of our general principle, in other applications, is almost certain to throw light upon the special applications in which we are most interested. To begin with, then, it is not only consistent with our theory of "prices," but is actually involved in it, that to any man, at any given time, there *may* be some alternative so horrible that sooner than accept it he would endure all the physical and mental torment that can possibly be inflicted on him. This does not necessarily mean that he does not feel the torture, though even that might be the case, but it means that the whole sum of torture which he is capable of enduring before his frame cracks will not be enough to overcome his shrinking from the only alternative open. Something must give way first, and if his resolve, or his aversion, is stronger than his physical vitality, the tissues of his frame will be disintegrated or his vital functions unhinged before his choice is reversed.

History shews that these conditions have from time to time arisen; and we contemplate with awe the heroes who have supplied the demonstration. We probably think that few people could rise to this pitch of heroism in any cause;

Are the
claims of
duty and of
faith exempt
from the
law?

but, on the other hand, it is no more than we have a right to expect of every normal human being, living a normal life, that there should be certain things which he would not do for any amount of money, however large; perhaps because he regards the actions as detestable or dishonourable, perhaps only because he regards them as intensely disagreeable. This only means that to him the total difference between the command of things in the circle of exchange that he already enjoys, and an indefinite or unlimited command of them, does not weigh as heavy in his mind as the dishonour or the discomfort of the specific thing that he is required to do. It does not mean that his objection is "infinite." It merely means that it is larger than his estimate of all the satisfaction that he could derive from unlimited command of articles in the circle of exchange, and this is a strictly, perhaps narrowly, limited quantity.

These considerations, it is true, do not completely satisfy us; for they would seem to imply that although the offer of money may not be enough to make an honourable man do a dishonourable action, yet if he is in want of money at all the offer must tend in the direction of making him do it, so that raising the bribe would strengthen the temptation. If it is true, as we have said, that every force tells for all that it is worth whatever other forces are already on the field, would it not follow that if a man is in want of money the offer of money must tell for what it is worth, whatever other motives actuate him? And if so, must he not be nearer to doing the dishonourable action (though he does not do it) than he would have been had the bribe not been offered to him? And if the bribe is raised (so long as he would still value the increased sum), must not the tendency to make him do the dishonourable thing become more marked? Or in the case of the martyr, if he shrinks from pain at all, must not the infliction of greater and greater degrees of pain tend to make him renounce his faith, though the inducement is not high enough actually to bring about the renunciation? It is true that there is nothing in these conclusions that greatly shocks our general experience or observation. We hear men say, "I confess I was almost tempted by the prospect, for a moment," or "It required all my resolution to hold out, I

can assure you," when they are speaking of actions the commission of which would have filled them afterwards with shame and self-contempt. But nevertheless we can by no means admit that every man can be at any rate tempted, though not seduced, by a bribe, or shaken, though not broken, in his resolution by torture. We are certain that this is not even approximately true as to the bribe, and we cannot believe that it is completely and universally true as to torture.

On this we may note, in the first place, that the very offer of the bribe or application of the torture may wake resisting forces which were dormant before.¹ I might be considering whether or not an action was really dishonourable before the bribe was offered, and as soon as a bribe is proposed I may have a conclusive reason for associating it with dishonour. Or again, if a man offers me half a crown for doing or saying something I may be contemptuously amused, but if he offers me £1000 I may be deeply insulted. For I might take the first proposal as a naive attempt to overcome my inertia, but the second as revealing a serious intention of finding out the price at which I would sell my honour. Thus the increased inducement might itself touch the spring of increased resistance. If the briber can contrive to associate his material offer not with dishonour but with some appearance of honour, and can make his insult take the semblance of a tribute of respect, it will perhaps be found that £1000 does indeed weigh more than 2s. 6d. in the scale. But even here a finer perception might detect the finer insult, and might resent it the more deeply for its deliberate subtlety.

But there is something deeper even than this, and its examination will lead us back to our economic and commercial investigations. Just as it is very easy to suppose that a man could tell the difference between a half-pound and a quarter-pound weight by trying them in his hand, but very difficult to suppose that he could tell the difference between 14 stone and 14 stone plus a quarter of a pound by lifting them in a basket, so it is very easy to imagine a man's refusing to give 1s. for a thing that he would be glad to have for 6d., but very difficult to imagine him willing to give £1000 for some object but refusing to

*The minimum
sensible.*

¹ Compare the qualifications to the Principle of Superposition on page 204.

give £1000:0:6 for it. That is to say, 6d. is appreciable when the whole matter at issue is only 1s., but inappreciable when the matter at issue is £1000. It is a case of proportion. When the stake is of any given magnitude there is a certain *minimum sensible* or minutest quantity that can be felt or appreciated in connection with it; and this *minimum sensible* will vary with the magnitude of the thing at issue. The same principle applies in the moral world. When my feelings are deeply moved and I am vividly realising any one of the main issues of life, things to which I should give careful attention on other occasions do not affect me in the least. The mind does not readily adapt itself at one and the same time to the higher and the lower end of the scale. When it is experiencing great things it is not sensitive to small ones. When some grave disturbance of equilibrium has occurred or is threatened, or some vast issue is at stake, small things are not felt. Only if the great things were secure and had not recently been disturbed would the small things be able to assert themselves as significant. If I hear of the sudden and unexpected death of a dear relative and immediately begin to speculate about his will, why am I ashamed of myself? Because I had imagined that my affection for him was so great that immediately on the news of his death the significance of a few hundred or thousand pounds would have sunk below the *minimum sensible*. And when I find that it is not so, I perceive that I have given myself credit for a higher appreciation of the things that are not in the circle of exchange, relatively to those that are, than I really possess. It is a startled sense of my own sordidness that brings my shame. It is not that I believe I ought not to care whether I have or have not the sum of money, but that I should have supposed that at that moment there would have been no room in my mind for such a thought, any more than for the fit of my trousers, or any other subject of consideration in itself perfectly proper but not sufficiently important to claim a share of my attention at the moment. I might experience the same kind of shock if, in catching up a child wounded by a passing dog-cart or motor-car, I found myself annoyed because my cuffs were stained or my clothes damaged by his blood. And the proof

that this correctly represents the psychology of the case is that if the question of the legacy or of the stained cuff merely presented itself to me externally but failed to touch the springs of interest or emotion, if it were a mere shadowy presence with no weight or "tactile value," I should note it as something strange, but should not feel it as anything shameful. The same analysis applies to occasions on which some great happiness comes to a friend accompanied by a slight incidental inconvenience or disappointment to oneself. The examination of such cases reveals the possibility of any given consideration sinking beneath the *minimum sensible*, but it also reveals the fact that in an enormous number of such instances the feeling or the motive that we neglect without one moment's hesitation is nevertheless actually felt. It is negligible, but if we look for it, it is there. It does weigh something, but it does not for a moment threaten to turn the scale.

Returning now to the martyr or the "incorruptible," we see that it is perfectly possible for the extremest pressure that can be brought to bear upon either to be quite negligible, so that it would no more be recognised as a reason (even an inadequate one) for doing the abominable thing than fear of staining my cuffs would be recognised as a reason against helping a wounded child. And it may be that it is not only negligible and practically unrecognised, but absolutely imperceptible even when we look for it. There is ample room for these facts within the limits of our theory.

Another point suggests itself for consideration in connection with moral questions. There is much confusion and ambiguity in our use of the word "duty." I may say that no personal or private considerations however urgent ought to affect the performance of my duty, even in the minutest point; but I shall not allow that I ought to leave a burglar despatching his business in my house rather than be a minute late at the office. "Of course not," it will be said, "because it is your obvious duty to protect your family, to say nothing of your property." Apparently, then, it is my "duty" to attend to whatever I conscientiously consider the most important matter at stake; and to say that nothing should interfere

Ambiguity
of the word
"duty."

with duty simply means that I ought to do the thing, whatever it is, which a high-minded man would regard as most important. Certain family claims which are not "duty" in a general way become so when they reach a certain point of urgency; and when satisfied down to a certain point they will again cease to be duty. In this sense "duty" is not a label which is attached to certain classes of action and not to others, giving precedence to the smallest volume of that to which it is attached over the largest volume of everything else. It is a name we give to the resultant course of action when every consideration has been given its due weight and no more, and nothing that is irrelevant has been allowed to weigh at all. And we shall generally find, on analysing any dilemma, that the dictum "Duty before all things" is only maintained by giving the name of "duty" to whatever, under the circumstances, properly comes first; and that our determination on this point is influenced both by the terms on which the alternatives are offered to us and by the extent to which we have already paid tribute to the one or the other claim. The label can only be attached after the conclusion is reached, and cannot indicate any short cut by which to reach it. If I insist on allowing no weight to any considerations that cannot be labelled "duty" in advance, I shall generally find that I must include in my "duties" not only my duty to my family and to my friends, but also that trump-card of the casuist, my "duty to myself." And I shall find myself speaking of a "conflict of duties," thereby implying that duty itself is a quantitative conception. It is of course true that if we are to allow no more than its due weight to a certain consideration we shall often allow it no weight at all, because it is irrelevant. If I am asked, for instance, to arrange a number of candidates in order of merit, I shall probably regard it as absolutely irrelevant to the matter in hand that a widowed mother is dependent on the success of one candidate, while another is a man of property himself and has no one dependent upon him, or that I am attached to one and am repelled by the moral character of another, or that I believe that success will react prejudicially on the character of one and favourably on that of another. And if I take this view, then undoubtedly it is my duty not

to give any weight to considerations that ought not to weigh, and it may or may not require some heroism on my part to act up to my convictions; that is to say, the temptation may tempt or it may not, as in the cases already noted. Or I may find that the real temptation is to incline to the verdict counter to my wishes, in order that I may escape the reproach of having been influenced by them. We may note that it is usual to protect examiners, as far as possible, from all knowledge of facts that are to be regarded as irrelevant; and this shews that the difficulty of ignoring them, if known, is generally recognised.

On the other hand, if I am making an appointment I may think that some or all of these considerations are relevant, and in that case it may be my duty carefully to appraise them all and weigh them against each other. When we have admitted that considerations of extreme strength in their personal appeal may be wholly irrelevant, and ought not to be realised as motives at all, even if they are felt, we shall have done full justice to the absolute conception of duty; but it is interesting to note how very many cases there are in which we are inclined at first to regard a consideration as irrelevant in principle, but find on close examination that a mere quantitative change in the things considered, if sufficiently pronounced, appears to us to raise the irrelevant into relevancy. In any case, our theory only asserts that when a consideration that "ought" not to weigh at all does as a matter of fact weigh—that is to say, is felt as a temptation—it may be felt more or less according to the magnitude and urgency of the issues at stake.

It is highly instructive to turn from the objection to the doctrine of declining significance which we have just examined to another which is quite as frequently urged. It is said that the whole theory of distributing our resources so as to gratify our wants *pari passu* and keep the marginal wants balanced, is false to fact and experience. The truth is, it is said, that there are certain things that we "must have," and we get "as much as we want" of them before we begin to consider less urgent requirements at all. For instance, we all eat as much as we want several times a day,

Do we secure
"as much as
we want" of
one thing
before we
secure any of
another?

and do not stop short of satisfaction because our desire for literature or travel is unsatisfied. Now to begin with, this is obviously an argument of the well-to-do. It is flagrantly untrue of the very poor that they get as much food as they want before they begin to trouble about keeping up their supply of clothes.¹ We have already spoken of the thousands of young people, well above the line of actual want, who in managing their own slender resources consciously and constantly bring their meal to a conclusion at a penn'orth or two penn'orth short of satisfaction in order to advance some other margin. In its crude form the whole contention that we are examining is palpably false. Where do we or can we find in civilized society the man who gets as much food as he wants "before" he gets any clothes or any shelter? All that can be seriously maintained is that if a man's resources are sufficient to provide him with a certain amount of the things he needs most urgently, including food, he will soon come to points in every other branch of his expenditure at which he will be content to rest until he has completely satisfied his desire for food as far as mere quantity, apart from quality, goes.

In the contention so formulated there is a great deal of truth, but it need not disturb our confidence in our general theory. Any one who has tried saving pence out of his meals by restricting them in quantity, not quality, will know that the significance of these pence rises very rapidly as they are successively withdrawn. A halfpenny-worth of bread (two thick slices of a half-quartern loaf) may carry a man from a sharp sense of hunger to a sense of satiety. To save 3d. a week on bread might involve a very considerable volume of unpleasant experiences, and therefore, unless the 3d. would minister (as in Cobbett's case) to very keenly felt wants in other directions, it would be bad husbandry to save it. "Yes," it may be said, "but by your theory to save 1½d. a week would involve less than half the sacrifice of saving 3d. a week, and its expenditure on something else would secure more than half the gratification of three pennyworth; and since by hypothesis the expenditure on bread is taken down to a point at which it ceases to have any significance at all, there must be some small quantity²

¹ Pages 34 sq.

² Cf. pages 66 sqq.

of the resources expended upon it that could be profitably turned elsewhere." This is theoretically true as far as it goes; but theory also tells us that this adjustment would be an exceedingly delicate matter, and that it might demand an amount of attention and exercise of will that could be more profitably employed somewhere else where it would have a higher marginal significance.¹

We have now examined two attempts to invalidate the general principle on which, as I have maintained, we administer our resources. It has been contended both that the sense of duty *ought to be* completely satisfied down to the last and minutest demand, and that the appetite for food *actually is* so satisfied, before anything else is attended to at all. The collocation of these two contentions is amusing; and before we leave them we may note that the sense of duty and the desire for food may become direct rivals. In that case I may perhaps cheerfully go without a meal at the call of "duty"; but presently I shall find that it has become my imperative "duty" to suspend the direct performance of my "duty" for a short time in order that I may eat something to enable me to perform my "duty" more strenuously (or to perform it at all) afterwards; and the graduated formulæ of "it is an imperative duty," "I almost think it is a duty," "I really think that without any dereliction of duty I may allow myself," etc., ease the (in this case) *difficilis descensus* from the pretentious heights of absolutism to the *avertus* (shall we call it?) of practical relativity.

Another and closely related aspect of the question of declining significances is suggested by charitable appeals. For instance, there is a famine in India, and I subscribe a guinea. That would appear at first sight to mean that I consider the want of food in India more urgent than any other wants of my own or any one else's to which the guinea would have ministered. But if so, why not give a second guinea? Has the want in India been sensibly reduced by my subscription? In bulk, yes. But in intensity? Even if I could suppose that my guinea had met the most urgent case, would there be any perceptible

¹ For a worked-out example see my *Alphabet of Economic Science* (London, 1888), pages 128 *sqy.*

decline of urgency in the next case waiting to be met? It is exactly the question of the increments of tea over again. We saw that there was no perceptible decrease in the significance of tea as we passed from one quarter-ounce to the next at the margin of 4 lbs., though there was a perceptible satisfaction in the consumption of either.¹ So I must suppose that a perceptible relief of suffering has been effected by my guinea, but I can hardly believe that a second guinea would relieve suffering perceptibly less intense than that relieved by the first. The marginal significance of a guinea, then, in relieving distress in India, appears to remain the same. Why do I not pay a second guinea and a third, and so on? The answer is twofold. In the first place, in the majority of cases it is not really the famine in India but my own conscience that I am appeasing, and my own conscience becomes perceptibly less clamorous after the first guinea has been paid. It may still grumble, and dispute the ground with other applications, but it may no longer dispute it successfully. My conscience may be right or wrong in insisting that I should take a share in the burden, and in being appeased when I tell it I have done so, but that is not the question. The point is that the demand I am meeting is, as a matter of fact, perceptibly reduced by what I have done to meet it. It is otherwise, however, if I really am directly appraising the urgency of the want that my guinea relieves when given to the famine fund, and the wants it can supply in other applications. In this case it is true that the want in India does not perceptibly decline as I give guinea after guinea, but it is also true that the wants that I neglect in order to meet it perceptibly rise as guinea after guinea is subtracted from the supply of them, until at last they rise to the level at which they balance my sense of the urgency of the need in India. This point may not be reached till I have reduced myself and all those dependent upon me to the level of misery of those that I am relieving; and some moralists are courageous enough to hold this up as an ideal. Our theory of marginal significance is elastic enough to adapt itself to their creed; for all that we assert is that, whatever the grounds on which we form estimates of the

¹ Page 54.

relative significance of rival applications of resources, we can so administer those resources as to bring their marginal significance in each application to equality. The urgency of the Indian claim is no doubt gradually declining if the administration of the fund is even approximately sound; but within the limits of the influence of my fortune it does not decline perceptibly. The balance is therefore found when all other expenditures are curtailed to the point at which their rising marginal significance equals that of the Indian claim.

Curious light is thrown on this class of problems by the added joy and relief which is not unfrequently felt by the recipient of a present that comes with the condition that it is to be spent on a holiday or on some personal indulgence. Presumably the recipient, if free, would have spent the sum as he wished. Why is he pleased at being forbidden to do what he would have wished? Because it is the sense of his duty to do the thing, not his sense of the importance of the thing's being done, that would have successfully contested the first place; and his "sense of duty" is entirely extinguished by the prohibition. The demand that would have had to be appeased before the other could be indulged is withdrawn from the lists, and the indulgence can be secured without a drop of gall. A goad has been blunted, and the hedonistic gain is obvious. In cases where this analysis would be untrue and where the wish to do something else with the money is really inspired by the eagerness of direct sympathy, the restriction would be actually felt, and perhaps resented, as a reduction in the value of the gift. Perhaps by the painful associations it waked it would altogether annul it or leave a balance to the bad.

Examination
of an apparent
paradox.

We have now concluded our examination of the class of objections to the law of diminishing psychic returns which is based on the absolutism of ethical or social conceptions; but in the course of these investigations we have been incidentally led to contrast a demand or craving that has to be appeased with an enjoyment that may be secured. This opens in its entirety the important subject of positive and negative satisfactions, their relations to each other, and the proper notation to be employed in their calculus; and to this subject we must now turn.

Positive and
negative
increments
and states.

If we regard pain as negative pleasure, and discomfort as negative satisfaction, then a supply of anything that gradually relieves me from acute suffering leaves me in a state of (decreasing) negative satisfaction throughout the process. But the reduction in the volume of this negative satisfaction, which is taking place all the time, is a movement in the positive, not the negative sense. It is an addition, not a subtraction, of desired effects: for it is a subtraction of undesired experiences. The acquisition, therefore, is a positive quantity, and must be noted by a plus, not a minus sign. Here we may introduce the familiar notation of curves. On Fig. 1 we measure the supply of any commodity per unit of time along the line OX , or the axis of X ; and on OY , or the axis of Y , we measure rates of satisfaction. Thus the curve

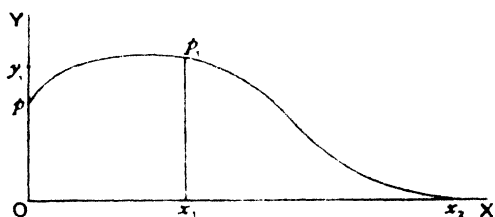


FIG. 1.

pp_1x_2 would represent that the initial increment of the commodity per unit of time satisfies some kind of desire at the rate of Op per unit of commodity; that by the time the supply is increased to Ox_1 the rate at which it is satisfying desire has risen to x_1p_1 or Oy_1 , and that when the supply reaches Ox_2 per unit of time, the desire is completely satisfied. The quantities measured along OX , which are called abscissas, indicate the breadth of the supply per unit of time, or the breadth of the stream of supply. Quantities measured along OY , which are called ordinates, indicate the marginal values investigated on pages 47-71 of Book I,¹ and areas such as Opp_1x_1 sums of satisfaction per unit of time, secured by the consumption per unit of time of the quantity of the commodity indicated by the corresponding abscissa. Generally speaking, such an area must (as we have here supposed) itself be taken as representing a rate of total enjoyment per unit of time, rather than a

¹ For the full justification of this statement, see below, pages 440 *seq.*, especially pages 446 *seq.*

sum of total enjoyment; ¹ but sometimes it will be convenient to take the whole figure as representing not a rate of consumption, but a single act. And in such cases we shall take x_1p_1 as representing the marginal value, and the area Opp_1x_1 as representing the "value in use" or total significance of the definite quantity Ox_1 . For instance, the figure might roughly represent the experiences of a single meal, during which for a time "the appetite comes as we eat" and we are conscious of increasing enjoyment, whereas after that point our hunger is gradually appeased to the point of satiety.

Now this diagrammatic method is useful as an instrument of research, as a means of demonstration and exposition, and, most of all, as a vivid and comprehensive form of statement. But it is very dangerous, and if not used with due caution and precision it may lead to grave confusion and may encourage loose and irresponsible thought. In the next chapter, accordingly, we shall examine the construction of one particular curve in great detail; and whenever we make use of curves we must try to bear in mind the necessity of giving an exact account of what they mean, so that the results obtained may not be in any way equivocal. The necessity for caution in this matter is illustrated on the very threshold, for (apart from the difficulty of determining how we are to measure a unit of satisfaction ²) we have to note at once that this first curve which we have introduced is ambiguous in relation to the very matter we are now discussing, viz., the relation between assuaging a craving and securing a positive enjoyment, or, more generally, between removing negative and securing positive objects of desire. We have seen that the removal of a pain must have the positive sign, and it must therefore be represented by a positive area, so that if we begin in pain and the supply of a commodity gradually removes that pain, the result must be represented as positive—comparable with, and to be weighed against, a gain of positive satisfaction. Our figure, therefore, will not tell us whether we begin in a state of positive satisfaction, a state of indifference, or a state of negative satisfaction, or pain. It will only tell us that if we command the quantity of the commodity represented by Ox_2 our state will be the *better*,

¹ Cf. page 101.

² See Chap. II. of this book, and cf. Chap. III.

by the whole area Opp_1x_2 , than it would have been had we had no supply at all. If we only command Ox_1 our state will be the better by the area Opp_1x_1 . The area $x_1p_1x_2$ will then represent either an unassuaged pain or an unrealised pleasure, but in either case the area Opp_1x_1 must have the positive sign. It is a gain, not a loss. The existence of the possibilities represented by the figure may in itself constitute a misfortune or a privilege; but granted their existence, the command of Ox_1 of the commodity, whether it means plus a pleasure or minus a pain, is a gain (in the estimation of the subject), and must be regarded as positive.

If we draw Fig. 2, it will represent the effects of the supply of a commodity which ceases to act in a positive

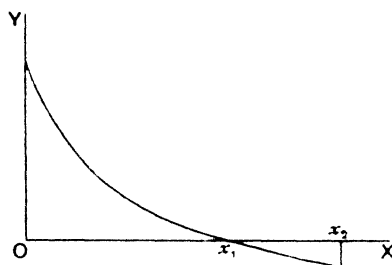


FIG. 2.

sense when it exceeds Ox_1 in quantity. Thus at a given temperature the consumption of fuel might begin by being extremely acceptable, and when it had reached the rate of Ox_1 per hour it might cease to be acceptable at all, and might, if raised still higher, become positively undesirable, or negatively desirable. Now one man may be so constituted that whereas he does not feel any positive distress by sitting without a fire, he may be conscious of a distinct pleasure if a fire is lighted; and another may be consciously miserable without a fire, and as the warmth increases may be conscious only of more or less adequate relief from discomfort till the quantity Ox_1 is exceeded, after which another kind of discomfort ensues from excessive heat. Yet another may at first be conscious of relief from suffering; then, before the quantity Ox_1 is reached, may feel that all his discomfort is gone and a positive enjoyment of the cosy warmth has succeeded to it; until, as the quantity Ox_1 is

exceeded, he feels that although the room is still positively pleasant it would be pleasanter yet if the fire were kept a little lower. To all these men alike the supply of the commodity up to the quantity Ox_1 will produce a result that should have a positive sign and should be represented by a positive area, though to one it is minus pain, to another plus pleasure, and to the third at first minus pain and then plus pleasure; and to all of them the further increments represented by the line x_1x_2 produce a result that should carry the negative sign and should be represented by a negative area, though to one it is plus pain and to another minus pleasure. All of them are in a state more to be desired as the supply grows from zero to

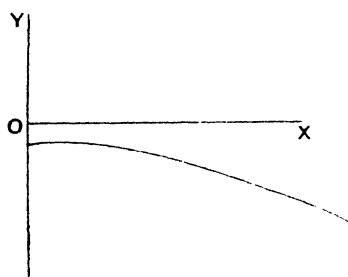


FIG. 3.

Ox_1 , and in a state less to be desired as it grows from Ox_1 to Ox_2 .

It follows from this example that an area below the axis of X , which represents negative satisfaction, may mean a subtraction from pleasure that leaves a positive balance, just as well as an addition of pain. Fig. 3 would represent a supply, or an experience, that, whether it detracts from the happiness of a happy state or makes a neutral one positively painful, or a painful one more painful yet, in any case produces a negative result, of increasing intensity per unit, as one increment follows another. If we are speaking in terms of positive satisfaction we shall still say that these increments have a declining (positive) significance, though if we were speaking in terms of negative satisfaction, or pain, we should say that they had a rising (negative) significance. Thus the fact that things which cause discomfort normally act with increasing intensity as unit is added to unit does not affect

the generality of our proposition that additional increments, after a certain point, produce decreasing (positive) results.

It sometimes happens that a positive quantity (in the technical and ambiguous sense in which it includes the subtraction off a negative quantity) is only to be had in association with a negative quantity. In that case probably the positive ordinates of the first will decline, and the negative ordinates of the second will increase, the movement in both cases being technically in the sense of positive decline. Thus a man who has bitten his tongue or has bitten a piece half out of his cheek may be in need of food, and yet eating may cause him acute annoyance. As his hunger or sense of faintness

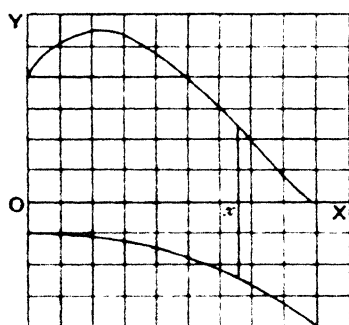


FIG. 4.

gradually yields, and his demand for food becomes less urgent, the increasing painfulness of the terms upon which alone he can assuage the declining urgency of his want will soon balance it, and his meal will come to what would else have been a premature conclusion. This might be represented either analytically by Fig. 4, or synthetically by Fig. 5. Both figures alike represent the fact that up to Ox an advance from the origin is accompanied by a balance of advantage, and that after that point the reverse is the case. And both figures agree in the magnitude of the advantage or disadvantage in either case.

Where there is no indication to the contrary a curve must be taken to indicate not a history but an anticipation, and an anticipation that has discounted (not necessarily for what they are worth) all conflicting elements, risks, and reactions as far

as they come within the ken of the person who makes the estimate. It will be a synthetic and resultant estimate of the balance of advantage to be anticipated from the acquisition of each successive unit of the commodity, of the type of Fig. 5.

We have noted that positive and negative quantities may be balanced against each other, and also that mathematically positive and negative quantities may both alike be ambiguous psychologically; for just as a subtraction from pain and an addition to pleasure are alike positive, so a subtraction from pleasure and an addition to pain are alike negative. Thus Fig. 2 (page 417), where the increments of the same commodity

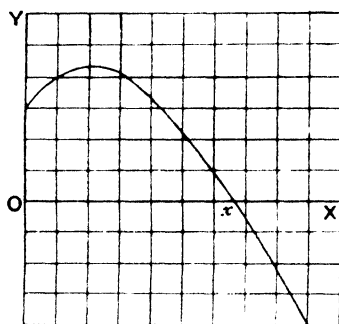


FIG. 5.

at first have a positive and then a negative effect, is explicit as to the positive or negative sense of the process in question, and as to declining (positive) significance of all increments after a certain point; but it is equivocal as to the positive or negative state of the person affected. He might be either in a state of suffering or a state of enjoyment throughout the process, or he might pass from suffering to enjoyment at any point on the line Ox_1 , or from enjoyment to suffering at any point on the line x_1x_2 ; but in any case he has either more enjoyment or less suffering as he passes from O to x_1 , and either less enjoyment or more suffering as he passes from x_1 onward.

Now, although the relief from a pain and the securing of a pleasure, or the deduction from a pleasure and the addition of a pain, have respectively the same signs, and

may be taken as equivalents, yet they are in themselves very different things. Given my constitution and circumstances, a certain relief from pain must be regarded as equivalent to a certain positive pleasure, a certain deduction of pleasure to a certain access of pain; and certain pleasures and pains taken together, or certain relinquishments of pleasure and escapes from pain taken together, must be regarded as balancing or neutralising each other; but it makes all the difference in life whether my constitution and circumstances are such that my energies have to be given chiefly to escaping or minimising undesired things or are mostly free for securing or developing desired ones, and whether I can often or only seldom get a pleasure without a concomitant pain or escape a pain without a concomitant loss of pleasure. And it is just here that our immediate choices react upon our future possibilities.

Difference
between re-
lieving pain
and securing
pleasure.

This subject of the reaction of our enjoyments, privations, and endurances upon our future capacities for enjoyment has already been touched upon in Book I.,¹ but the investigation we have just completed will now enable us to enter upon it more fully. We have to make constant adjustments between the immediate gratification of desires and the building up of capacities. A great part of wise conduct obviously consists in forgoing a present gratification, or incurring present pain, or making irksome effort, in order to acquire a capacity for future enjoyment, or power ultimately to secure or promote desired ends. Wise administration of vital resources must therefore take constant note of this reaction of the present upon the future.

Every wise man must desire to build up for himself such habits of mind and body from within, as well as to surround himself with such outward circumstances, as will make life as little as possible an escape from wretchedness and as much as possible an experience of well-being and an achievement of desired ends. We must therefore cultivate the power to endure such undesired experiences as are inevitable, and to forgo such desired experiences as are unattainable, with the minimum of suffering, and to derive the maximum of satisfaction from the realisation of things desired. An

¹ See page 85.

example may make this clear. Two men are on a tour together in a beautiful and sparsely inhabited country. They find themselves out of their reckoning, and when dinner-time comes they are far from any opportunities of dinner. The spirits of one of the companions begin to sink, his temper becomes unstable, he cannot enjoy the scenery through which he is passing, the exhilaration of mountain air or of the battle with the waves is a thing he knows not, the suggestion to turn aside and spend half an hour in ascending a rock or exploring a cave is fiercely resented, and, in fact, the man's whole moral, æsthetical, and physical being is swept up into one hideous craving for food. At last the friends (if they still deserve the name) reach hospitable quarters. Their hostess wishes to do justice to her reputation and keeps them waiting for an hour in order to set a noble repast before them. But when it comes it is too late. The poor wretch can now eat nothing, and goes sick and miserable to bed. His companion (so far as his sympathetic heart allowed) has meanwhile been drawing in delight at every pore, keenly enjoying the tussle with the waves or the stride across the heather, with an eye that (like Wordsworth's) finds no hairbreadth of earth, sea, or sky from which it does not gather delight, ready at any moment to turn aside and delay the end of the journey in order to increase the enjoyment of its progress, conscious indeed of keen hunger, but conscious of it rather as a prospect of future pleasure than as a present experience of pain; and when at last he finds himself opposite his victuals, a harmony is established between the organism and the environment which almost rises to the dignity of a spiritual experience. The less fortunate of these travellers derives the maximum of suffering and the minimum of enjoyment, the other the minimum of suffering and the maximum of enjoyment, from the necessity of taking food. The one is the victim of a craving; the other has a capacity for enjoyment. To the one it is agony to be thwarted, and only a negative satisfaction to be humoured; to the other privation is no pain, but a supply "adds sunshine to daylight."

The wise or happily constituted man has a mind so regulated that many of his desires only become rampant as

the prospect of satisfaction approaches. Till then they are dormant potentialities of enjoyment. Thus the man who on coming in sight of a public-house declared that he "had a thirst on him for which he would not take £5" was perhaps to be congratulated if he had been thoroughly happy before he saw it; but if he had been miserable himself and a cause of misery to his companions for the last hour or two because there was *not* a public-house in sight, he was an unenviable person as well as an undesirable companion.

What, in the instances we have given, may be regarded at any rate primarily as a difference of physical constitution has all manner of analogies in acquired habits of mind and body; and every wise man would desire for himself and others such habits and impulses as would conform to the happier type. Now, though all means or opportunities of gratification seem to have this in common, that the immediate effect of successive increments is (after a certain point) of declining positive value, yet different kinds of gratification differ enormously in their after-effects upon the organism itself. Is our present enjoyment building up an increased capacity for future enjoyment? Is it leaving us permanently unmodified, so that after a time we shall return to exactly the same state in which we were before? Is it undermining our power of future enjoyment, so that after every act of indulgence we return not to the same, but to a lower power of enjoyment than we had before? Or is it substituting a craving for a capacity for enjoyment?

The characteristic of ruinous enjoyment is that it not only tends to satisfy us at the time (as do all enjoyments), but that it also tends to undermine our capacity for future enjoyment. The most pronounced forms of ruinous enjoyment are probably those which are popularly regarded as vicious, such as intemperance. The characteristic of a vice, from a hedonistic point of view, is that it tends to replace a capacity for enjoyment by a craving. Intoxication may be extremely delightful, but the more habitually a man drinks, the less pleasure it gives him to be drunk and the more pain it gives him to be sober. He begins, perhaps, by hitting on a means of heightening enjoyment; but he ends by being

Building up
capacities for
satisfaction,
or desired
habits and
impulses.

Ruinous
satisfactions
and vice.

in a state of chronic misery, from which he gains occasional respite in an intoxication which no longer gives him any positive pleasure. His whole conscious being has been swallowed up in the vortex of one frightful and incessant craving. This is a typical case of ruinous enjoyment. I am not here concerned with any attempt to analyse the ultimate grounds of the reprobation implied in the terms "vicious" and "vice," but it is interesting to note that the popular moral judgment stands in intelligible relation with the results of a hedonistic calculus. And note that our diagrammatic method gives us no notice of this change from a source of pleasure to a craving. Diagrammatically the appeasing of a craving is indistinguishable from the securing of a satisfaction; and if the acquired craving is more imperious than the natural desire for pleasure originally was, we should have to represent the change by an increased height of the curve indistinguishable from the representation of an increased capacity for enjoyment.

But there are many enjoyments which, so far from producing a vicious craving, rather tend to beget a sense of satiety, or even disgust, unless kept within very moderate limits. The danger here is not of converting a possible source of enjoyment into a craving, but simply of deadening by indulgence the susceptibilities from which the enjoyment springs. For example, most people enjoy a little salmon occasionally, and are inclined to regard it as something of a treat; but it is pretty generally known that, if used as a staple food, salmon very soon loses its charm. The provision long customary in the indentures of apprentices, that they must not be required to eat salmon more than so many times a week, is the historical record of this fact. Salmon therefore could not well take the place of the Englishman's traditional rasher of bacon as the breakfast dish for all the year round. It seems to be a fairly general experience (though of course by no means universal) that you may eat fried bacon for breakfast whenever you are inclined to do so, and may continue to be so inclined day after day and year after year; whereas if you were to eat salmon whenever you were inclined to do so, you would very soon cease to be inclined to eat it at all. The appetite for bacon, then, when extinguished for the moment, rapidly recovers its pristine

Wasteful
satisfactions
and luxury.

vigour; whereas the appetite for salmon, unless it is allowed a long period of recovery, becomes permanently lowered or deadened. If a man, though eating salmon as often as he feels inclined, does not eat as much at a time as he is inclined to do, the effect may be deferred. But even so, salmon will soon cease to be much of a treat.

Again, a man is not likely to eat oatmeal porridge for the pleasure of the palate when the appetite (as an index of an organic demand of the system) is assuaged; whereas the skilled cook, "by successive intensifications of his diabolical art," may tempt a man from excess to excess by appeals to his palate, even when his appetite has long been sated. Now

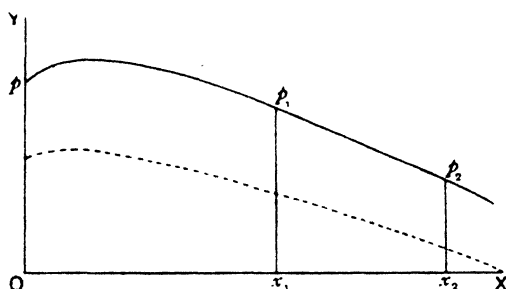


FIG. 6.

healthy and vigorous persons who are accustomed to simple and frugal ways are perhaps conscious, or subconscious, on most days that they would enjoy a rather more elaborate diet than they are accustomed to. But every one who has had experience of the two ways of living will tell us that those who live with severe simplicity get more enjoyment out of their meals than those who have an elaborate dinner every day. It is very easy to see why. The man who tries to extract the maximum of sensuous satisfaction out of every meal is securing trifling increments of satisfaction at the margin to-day, and is thereby deadening his capacity for enjoying the more significant increments nearer the origin¹ to-morrow. He is not indeed substituting a craving for a source of satisfaction, but he is lowering his possibilities of satisfaction. Thus, if a man has a moderate supply of any such luxuries as we have been discussing, his enjoyment may be represented by Fig. 6.

¹ "Origin" is the technical term for the point marked O in all our figures.

He stops at x_1 , and there are still unexhausted possibilities of enjoyment. But if he habitually goes on to x_2 , though at first he secures the additional area of enjoyment $x_1p_1p_2x_2$, yet he gradually lowers the significance of the initial increments, and ultimately only enjoys the smaller area bounded by the dotted line above Ox_2 instead of the larger area Opp_1x_1 . Again, the man who eats or drinks as soon as he is inclined to do so, often falls into the habit of eating and drinking as soon as he is able to do so; and, as he never recovers a state of healthy hunger, he too always remains at the low level of enjoyment.

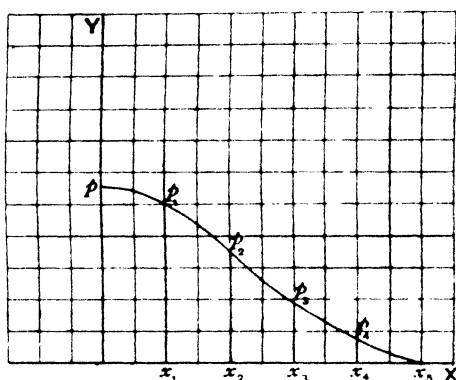


FIG. 7.

Let us take another illustration. Some moderate smokers will declare that a pipe two or three times a day gives them great satisfaction, but that they do not miss it, in the sense of feeling any positive discomfort, if for any reason they are deprived of it. For the time being a single pipe completely exhausts the possibility of enjoyment, so that they would find no pleasure in further smoking. Let Fig. 7 represent the total pleasure, declining from the initial point of intensity to the point of complete satisfaction.

It is obvious that after a pipe has extinguished the present possibility of further enjoyment a certain time must elapse before it is recovered; and it will not be recovered suddenly. Let us suppose that after an hour the area of possible enjoyment $x_4p_4x_5$ has been recovered; that is to say, the man is in the condition in which he was when he had smoked four-

fifths of his pipe. He may now enjoy a cigarette that contains one-fifth of a pipeful of tobacco as much as he enjoyed the *last fifth* of his pipe, and if he repeats this every hour he enjoys five times the area $x_4 p_4 x_5$ in the course of five hours. Whereas if he had not smoked for five hours he would then be just where he was before he smoked his last pipe and could enjoy the whole area $O p x_5$ again.

We have seen that our diagrams do not distinguish between the assuaging of a craving and the conferring of a positive satisfaction, and that in many cases the earlier increments of a commodity may perform the first function, and the later increments the second; and, moreover, that the two may overlap. In the case of smoking it is possible, though not usual, for a man who enjoys it to be able to abstain completely from it without positive suffering. In the case of food or drink this is impossible. Thus, if a man had a suitable allowance of food and drink, he might divide it up into a number of rapidly succeeding nibbles and sips (like cigarettes), or he might take larger portions at longer intervals. It would seem that in such cases the man who does not allow his organism time to recover its full sensitiveness to pleasure before he endeavours to extract renewed enjoyment out of it, and the man who pushes abstinence to the point of positive pain and craving before he assuages it, supposing them both to eat the same amount, would be alike wasteful in their administration. The man who lets his organism recover its power of yielding enjoyment without inflicting positive suffering on it (or, if the two states overlap, goes back to the point at which the pain incurred and the pleasure secured just balance) is administering his resources to the best advantage.

Self-indul-
gence and
asceticism.

Note here again the extreme care that must be taken in the use of diagrams. If our curve in Fig. 7 represented the value of successive increments of any commodity per month (as in the case of tea in Book I. Chap. II.), or per year, or per day, it would take no note of the different effects of the same rate of supply differently distributed within the period in question, which is the problem we have now been discussing. Some system as to this internal distribution is tacitly assumed (as it was in our former tea problem) as constant during the

whole inquiry, or as modified according to some consistent system as the supply contracts or expands. This is as it should be, for whatever particular condition we are examining and are supposing to be subject to variations, it must always be assumed that the other conditions are constant.

To return to our main inquiry. We have seen that certain kinds of enjoyment, and certain habits of consumption, while apparently innocent in themselves, are eminently wasteful from the hedonistic point of view, either because they more or less permanently deaden the keener powers of enjoyment, or because they never give those powers the opportunity of recovering themselves. And yet deliberately to stop eating salmon when you would like more, in order that you may be able to get more pleasure out of a help of salmon this day week, is a piece of self-conscious sybaritism from which the healthy mind revolts. Even the man who will not eat when he is hungry and has suitable food before him, for fear of "spoiling his appetite" for a more sumptuous repast which he expects in a couple of hours, fails to excite our admiration. We seem then to be in the presence of a kind of waste against which it is impossible to provide without unworthy attention to appetites that are only wholesome so long as they are unreflective. And so indeed we are. But our analysis has resulted in a triumphant vindication of certain instincts which we may henceforth trust more completely, and which, if we follow them, will effect the desired saving and give zest and vigour to life, without any habitual self-consciousness. Luxurious living has always lain under suspicion as hostile to a vigorous life, as something which, if not absolutely culpable, deserves a certain disapproval, and moreover as self-defeating even on its own chosen ground of physical enjoyment. Self-indulgent habits which, on the face of it, only seem to open up innocent sources of enjoyment are nevertheless regarded with a certain contemptuous impatience by healthy and vigorous minds. The man accused of self-indulgence retorts on his critic with a charge of asceticism; and his mentor, while repudiating the charge, often find it difficult to defend by logic the position to which he is guided by an obscure instinct. But that obscure instinct, we now see, is perfectly sound, and it warns us against

forms of enjoyment which, if not viciously ruinous, are yet wasteful.

We seem now to have got at something like the philosophy of it. The self-indulgent person is perpetually nibbling and never giving himself the chance of a hearty meal. The ascetic is always cutting back to the point at which the potentiality of a satisfaction passes into the realisation of a pain. And both alike debilitate their frames, and unduly concentrate their minds upon material sources of satisfaction. For, be it observed, persons who have practised genuine asceticism (as distinct from persons who by nature or training have become indifferent to what most men enjoy) will generally tell you that they were never so greedy in their lives as when they fasted severely; and perhaps that they have never quite recovered from the effect of the practice. A sufficient effort of will, or a strong enough preoccupation, may extinguish or indefinitely suspend a craving, but to maintain a want at the stage of craving, without extinguishing it, is to fix the mind upon it. Hence many curious parallels in the moral effects of luxurious and ascetic living; and hence the justification of the instinct for a robust and simple life that shuns both.

We can now fully understand the recognised failure of all elaborate attempts to make life enjoyable by luxuries. A rich man trying really to enjoy himself in the midst of his wealth often suggests a man attempt-
Hedonistic
value of a
simple life.
 ing to bathe in his Sunday clothes. He cannot feel the sweep of wind and water over his limbs. Hence the genuine but futile wail of persons surrounded by luxury, hence their craving for the "simple life," and their restless longing to break away from their surroundings and to put themselves into circumstances where money positively will not command any but the simplest supports of life. Only so can they get into contact with the initial satisfactions which are reserved for those whose nerves have not been deadened and blunted by being called upon to respond to fresh supplies before they have recovered from the last, or to seize a little more excitement at the margin to the detriment of their tone at the origin. There can be little doubt that those who constantly go without things, not

because they do not want them, but because they cannot get them, and who have an unfailingly abundant supply of nothing but a few simple things, selected by experience for their staying qualities, get more physical enjoyment out of life, and a larger amount of physical delight out of their contact with things, than all the devices of luxury can secure. And, very happily, this mode of ordering life, with all its invaluable reactions, may be maintained, when once deliberately embraced, not by thinking but by not thinking about it. The man who cares most for other things will act with the greatest wisdom in these matters; and he will instinctively form habits, or, if you like, contract prejudices, which without self-consciousness will secure the best fruits of reflection.

This question of self-consciousness enters closely into another problem, which has to be faced in all housekeeping above the lines of poverty and below the lines of luxury. We have seen that "second helps are never so good as first," and it would seem to follow that there is a *prima facie* gain (under the reserves indicated on pages 82 *sq.*) in having no second help to-day, but another first help to-morrow or this day week. That is to say, if green peas or new potatoes (in themselves, let us take it, of the "staying," not the "cloying" order of commodity) are a treat which cannot be indulged freely, it would seem to be better to have a little often than a great deal seldom. And many housewives follow this line. But it is by no means above challenge. Children who are habitually stopped at the first help when they keenly desire more will almost certainly become greedy, if the reason given for stopping is that they may have the rest to-morrow; whereas if they had sometimes had as much as they wanted, and none at other times, they might have remained healthily animal. And so we are back again at the point which we encountered early in our inquiries.¹ We may pay too heavily for securing the best possible administration of certain defined resources in their application to their immediate purposes. On the whole, may we not say that the popular instinct regards as the most desirable life one which is simple to the verge of severity,

The perfect
Epicurean.

¹ See pages 21, 82, etc.

but which allows a certain amount of variety, and prefers long or even complete and permanent abstinence to stinted and watched indulgence? Bread and water, Epicurus declared, were good enough for him; but for all that he would like a bit of cheese, so that he could have a blow out when the fancy took him. We may be sure that when he did have cheese he liked to have plenty. I once heard of a servant girl who every year bought and cooked for her single self a peck of green peas. She said she liked to "have her fill o' peas" once a year, and when that was accomplished she was in a state of equilibrium for the rest of the season. She was a true Epicurean.

As far as material indulgences are concerned, then, the instincts of popular moral judgment condemn the most ruinous forms of enjoyment as vicious, regard less ruinous but still wasteful forms as undesirable, if not exactly culpable, and look askance at too scrupulous attempts to economise and maximise enjoyment, as savouring of self-conscious materialism and wanting in directness and robustness. The man who so orders his life that, with small or great variety, he periodically pursues his enjoyments down the slope of diminishing returns to a point determined by his general resources and the claims upon them, but never dulls his capacity for periodical renewal of them, escapes the censure of the most rigid moralist. He is "living the simple life."

But there is another kind of satisfaction, the indulgence of which positively increases the capacity for future enjoyment. The man who enjoys himself in such ways as neither to reverse nor to destroy nor merely to maintain, but to increase his hedonistic capacity, gets a curious kind of credit for his conduct. Intellectual, literary, and artistic enjoyments (to those who really enjoy them) belong to this class. Most of them demand at some period or other a certain more or less painful effort and discipline.

Probably no one can get the highest and most sustained form of enjoyment out of literature without a considerable amount of drudgery of one kind or another; and the same is true of art, and at least equally so of science. Even exercises or studies which are in the main enjoyable must often be

Capacities
for enjoy-
ment which
indulgence
develops.
Painful
training of
capacities.
"Superior"
tastes.

pursued all down the scale of diminishing returns of satisfaction until they cease to give any pleasure at all and become in various degrees painful, if we are really to make anything of our studies. Some wise man (is it Ruskin?) has said that if we wish to do our best we must never work against the grain, but if we wish to do better than our best we must often go on when the work is irksome. We shall spoil it, but next time we shall do better than our former best.

Now this kind of gratification, sometimes merely pursued past the point of enjoyment, sometimes associated with painful training or irksome preparation, but always tending to create an increasing fund of possibilities of enjoyment, is regarded by the popular instinct as "superior." We speak of people who cultivate such sources of satisfaction as having "superior tastes." The slight half-veiled contempt for the "superior" person that we can often trace is apparently due, partly to a doubt whether he really does enjoy his superior pursuits, and partly to a suspicion that he may be starved into them by the lack of a wholesome and vigorous appetite for the robuster enjoyments of his neighbours. Lady Jane Grey appeared to prefer reading Plato to hunting and hawking; but did she really prefer it, or did she only wish to prefer it, or wish to be thought (by herself and others) to prefer it? And if she did prefer it, was it because she got more out of Plato or because she got less out of hunting and hawking than the others did? Was it the presence of a faculty they had not, or the absence of a faculty they had, that made her choice differ from theirs? Our respect for "superior" tastes when they are genuine is shewn by our extreme desire that the "working-man" should contract them, by our distress if more fiction than history and science is taken out of our public libraries, and our willingness to bear a part of the expenses of lectures on "superior" subjects—for others to attend.

Roughly speaking, these more fruitful enjoyments seem as a rule to be less exclusively and often less directly connected with the senses than the neutral or ruinous enjoyments are. It is true that the eye and ear are directly concerned in the enjoyment of music or of art, but the element of intellectual analysis and judgment, and, far more, the element of imaginative and emotional association, play a preponderating part

in them. In the enjoyment of literature or of scientific investigation the place of the senses is still more subordinate. Now it is generally regarded as an axiom that mental and spiritual enjoyment is of a higher order than the enjoyment of the senses, and it is interesting alike for those who are, and for those who are not, prepared to receive such a judgment as axiomatic, to note that at any rate it finds itself, like the other moral judgments we have examined, in easily traceable relations with the hedonistic calculus.

But the coincidence is not quite complete. For capacities that can be developed and rendered fruitful, perhaps at the expense of initial pain, sometimes yield material, not spiritual or intellectual satisfactions. They are then on a level with "superior" satisfactions hedonistically. But the moral judgment declines to consider them "superior." The process of learning to smoke wakes no moral enthusiasm even if it results in a power of enjoyment free from any vicious or wasteful craving. Having the ears pierced for earrings, in the old days, was only regarded as really praiseworthy by those who thought it a woman's first "duty" to make herself attractive. No one gets moral credit for what has been called "the long and painful apprenticeship to the art of liking olives." We have got some light, I trust, in this chapter on the relations of instinctive moral judgments and the results yielded by a hedonistic calculus; but it is far from my own belief that the one can be completely resolved into the other. This last set of instances may serve as a warning against any such belief.

The relation between popular moral judgments and the result of a hedonistic calculus.

The tendency, not fully accounted for by hedonistic considerations, to attach a note of intrinsic inferiority to pleasures of the sense is curiously illustrated by the case of connoisseurship in wines. If an interest in wines and a delicate judgment of them is combined with strict moderation it presents many of the qualities of an artistic enjoyment; and the old-fashioned elaborate conversation about wine presented a curious analogy to the discussion of the merits, say, of pictures. Yet to have given such close and earnest attention to things of sense suggested a more or less material view of life. Hence a somewhat confused feeling. Connoisseurship in wines seemed

in itself to belong to a "superior" order of enjoyments, but by its associations and suggestions, to an "inferior" order; and accordingly it often provoked in the mind of the impartial outsider curiously mingled and conflicting feelings, now bordering on contempt, and now rising to something very like respect or even envy.

It will hardly have escaped the reader's notice that our examination of the reactions of different enjoyments upon the organism, and especially the section on the waste-fulness of enjoyments of the intrinsically cloying order, or enjoyments carried to the cloying point, has been a running commentary on the dangers of civilisation and of increased command of material comforts. If wisdom does not grow with power, our latter state, even from the material point of view, may well be worse than our former, as material wealth increases; and the action of the economic forces, unguided and unchecked, naturally favours the growth not only of a class of ministers to vice, but of a class of persons who live by enabling people to get another drop out of the squeezed orange of to-day's capacity for enjoyment, reckless of its reactions upon to-morrow. And further, it will be seen that the "simple life" comes, if at all, rather incidentally as a natural result of caring for worthy things than as an object self-consciously aimed at for its own sake. The remarks on pages 186-189 may be re-read in the light thrown on them by this chapter.

Nothing that has been said in this chapter must be taken as committing the author to a hedonistic theory of ethics.

Hedonism not involved in our general principles. Suppose a man deliberately desires to cultivate impulses, and to train himself to a sense of values which he does not expect to give him the maximum of personal happiness. Suppose there are things that he really does care for more than his own happiness, or impersonal objects that he wishes he did care for, and hopes he one day will care for, more than for his personal enjoyments. Such a man would endure suffering, sacrifice pleasure, and fight against many of his impulses, in order to secure a permanent set or habit of will and a firmly established scale of values which could only be justified by reference to some social or religious test. These purposes would have secured his loyalty,

but not on the ground that they promised to secure his happiness. But the formation of such habits and the cultivation of such affections would, in this case, be the man's active desire, for whatever reason; and he would sacrifice the gratification of other desires in pursuing it. His self-discipline and his renunciations would be, from our point of view, of the same order as those of the man who undergoes irksome discipline for the sake of acquiring a hedonistically valuable taste, though he would not be moved by hedonistic considerations. It is not my purpose, however, to discuss ethical theories, but merely to shew that the general principles on which our investigations are based, while throwing light on the hedonistic calculus, do not presuppose a hedonistic theory, but are equally applicable to any other.

I will conclude this chapter with a few additional notes on the nature and limitations of the diagrammatic representations we have used. They may be best regarded as attaching themselves to the examination of roused and dormant desires on pages 422, 423. A large number of personal curves probably rise for some time before the ordinates reach their maximum and begin to decline. The matter is a little difficult to decide, for it is not easy to keep it clear from the considerations, entered upon above, of changes in the ethos of the individual during any considerable period. But it may well be that the same man with the same tastes and capacities would be willing to pay a larger sum for, say, a second chance in the month of hearing good music than he would for the first, possibly more for a third chance than for a second (and then less for a fourth and fifth, and so on), not because his musical taste is improved, but because his musical appetite is roused. In any case, when a dormant capacity or desire is roused, or a mild one stimulated, an abrupt or early cessation of the means of satisfying it may leave us in a balked or aching state, which constitutes a pain in excess of the original sense of want or privation (hunger, or what not) which is as yet imperfectly relieved. It is possible that, starting with any given condition, and regarding relief from discomfort and positive pleasure alike as positive, the sudden arresting of satisfaction might leave a legacy of actual pain which would not be represented on our

Limitations
of the
diagrammatic
method.

diagram; because the supply of the commodity has a positive value as long as it lasts, and would continue to have a positive value if it proceeded. Fig. 8 might give some kind of representation of such a case. It might mean that the man started from a state of indifference, but pursued some occupation or enjoyment with growing keenness, and derived a pure access of satisfaction as the appetite was at once roused and gratified. Up to the amount Ox_1 he has secured the area of satisfaction Op_1x_1 , and there remains an unexhausted possibility of satisfaction represented by the area $x_1p_1x_2$. But if the supply is now broken off, the unsatisfied desire continues and the satisfaction ceases. The result is a pain represented by the negative area

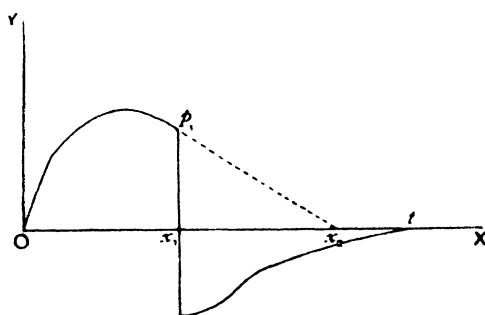


FIG. 8.

below x_1t . It is only after a lapse of time represented by x_1t that the pain wears itself out and the man returns to his initial state, having experienced both a positive and a negative satisfaction, the latter of which might in some cases be the greater. In such cases we say we had rather have had none of a thing at all than the tantalising amount we secured, even though we thoroughly enjoyed that little while it lasted. Fig. 8, however, is a monstrosity; for progress along the axis of X means increments of commodity up to x_1 , and for the positive area above, up to x_2 ; whereas for the negative area it means the passage of time from x_1 to t . It is really two figures, and the units of area alone are common to the two.¹

Returning to the phenomenon itself, we note that it may occur in every case of gratification arrested short of complete satisfaction. As a rule we may suppose that the lower the

¹ Cf. page 441, and the whole of Chap. II.

point to which we have reduced the ordinate the smaller will be this offset of dissatisfaction. And in a well-filled life it will often be absolutely eliminated; for although the lowest increment of satisfaction has not been squeezed out of some indulgence, and a theoretical sense of want might supervene if the next occupation or experience of the man were inherently neutral, yet if there is some other pleasant or desired occupation to which to turn, the anticipation of it substitutes eagerness for something else in the place of a languid desire to continue the present experience on the declining slope. Perhaps the best theoretical defence of smoking that has yet been discovered by the numerous and able advocates engaged in the cause is the assertion that it prevents listless and self-indulgent persons from over-eating, because when the keen demands of appetite have been satisfied but there is still enough left to dally with, the seductive prospect of a smoke turns the mind into another direction and offers a greater satisfaction from the arrest of the process of eating than can be gained from its continuance.

It is a fact pointed out and abundantly illustrated by the psychologists, that the very same present sensations may be pleasant or painful, according to the anticipations of the immediate future with which they are associated. The hunger that is a conscious pain, if the prospect of a meal is at all remote, may be a source of keen pleasure to the man who actually has his victuals before him, even before he has eaten the first mouthful. And in the same way the man who is accustomed to associate self-control with vigour, enjoyment of life, sense of command, and self-respect, may derive positive and immediate satisfaction from the absence, at the end of every meal, of that "sense of repletion" which in itself, according to Alexander Bain, is "massive and serene."

The conclusion of the whole matter, so far as our diagrams are concerned, is that it is generally an abuse of the diagrammatic method to attempt to make a curve represent, with any closeness, an isolated and concrete experience. A curve must represent the *estimate formed by the consumer* of the value to him of the successive increments of the commodity, and that estimate will be formed in view of all the immediate effects

and remoter reactions and implications which he is capable of appreciating. All these considerations therefore will tell on the height of the ordinates, which must be regarded as registering the resultant estimate. The anticipations on which they rest will perhaps never be perfectly justified; but as anticipations they have already made all the necessary discounts, and they need no kind of supplementing or correction. Declining ordinates mean that the consumer, taking at his own valuation all the considerations that can influence him, desires successive increments of the commodity with declining eagerness; and his estimates are based upon anticipations which are constantly being checked and modified by experience.

CHAPTER II

ON THE DIAGRAMMATIC METHOD OF REPRESENTING AREAS OF SATISFACTION AND MARGINAL SIGNIFICANCES¹

SUMMARY.—*The method of representing economic phenomena by curves demands closer examination than we have yet given it, and turns out on inspection to present many problems both of interpretation and construction. The measurements on the axis of Y indicate limiting rates of marginal significance, and, while expressed in an objective rate-unit, they must ultimately rest on estimates based on psychic experience. Hence difficulties arise as to the relation between objective and psychic units, the possibility of keeping that relation stable, the meaning we are to attach to accuracy of estimate and the conditions which limit that accuracy. If we express the data of Book I. Chapter II. as to the significance of tea in the form of a tea curve we are led to examine (a) the implications of the special formula to which our data conformed, and (b) the possibility of any simple mathematical formula approximately representing the facts. An attempt accurately to interpret the curve further leads us to distinguish between a curve of total satisfaction and marginal significance on the one hand, and a curve of price-and-quantity-purchased on the other hand. We find that these curves can, at best, only coincide approximately, and that an individual curve purporting to represent both series of phenomena can theoretically only be a "temperamental" compromise.*

In the preceding chapter I have represented satisfactions

¹ Chapters II. and III., though important from the theoretical point of view, are of an abstract and somewhat academic character, and some readers may prefer to go on at once to Chapter IV.

by areas bounded by curves, though with the express reservation that this procedure raised questions and required explanations upon which it was not convenient to enter at the time. We will now proceed to a more careful examination of this method. We shall frequently employ it hereafter.

The representation of a given satisfaction by an area of any kind, whether rectilinear or curvilinear, involves by implication the conception of a unit to which different satisfactions can be reduced, and in which they can be expressed for diagrammatic comparison with each other. And though this idea is far from familiar and presents great difficulties when first expressly suggested to the mind, we have nevertheless seen that it is directly implied in all our practical dealings and deliberations; and it underlies all the investigations upon which we have hitherto been engaged. For to say that two things are of equal value to us, and that another thing is just as valuable to us as both of them put together, is to say that the latter is worth twice as much to us as either one of the former, or that we anticipate a satisfaction twice as great from the one as from either of the others. If we say that a thing is just worth a penny, we are thereby equating the satisfaction we expect it to yield with all the other satisfactions which we believe a penny would secure at the margins of other branches of expenditure, and if we went on to say that something else was worth exactly three shillings and not a penny more, we should be saying that we expect it to yield as large a satisfaction as any thirty-six things we could get for a penny each, or a satisfaction thirty-six times as large as that which any one thing just worth a penny is expected to yield. Now it is quite true that such estimates are often vague, and almost casual, and that they are subject to every kind of fluctuation and inconsistency; but every deliberate act of choice, or of administration of resources, is an attempt to make them more precise and consistent; and even an impulsive choice is a declaration that at any rate one thing is more valued by us than another, and this involves an act of quantitative comparison. Such as they are, these choices, impulsive or deliberate, are verdicts as to comparative volumes of satisfaction, considered

as magnitudes, and they often express themselves in units of pence and shillings.

Now all commodities, services, or opportunities that enter into the circle of exchange are ultimately estimated not as physical or objectively measurable magnitudes, but as sources of anticipated satisfaction; and we frequently estimate things that are not in the circle of exchange in terms of things that are, and constantly choose between things that are and things that are not in this circle, weighing them against each other. Thus it is clear that for each one of us, at any given moment, the ordinary conduct of life unmistakably implies and involves the conception of satisfactions as magnitudes, and therefore as expressible ideally in units, which may be represented diagrammatically by unit lines, or areas, or otherwise, as suits our convenience. And just as, in measuring and comparing lengths with a view to determining their relative magnitudes, it does not matter whether our unit is an inch, a metre, or a mile (the difference being only in the numerical expression of the results obtained, not in the results themselves), so it is of no consequence whether we take our unit of satisfaction as that represented by 1d. or that represented by £1. But in comparing different satisfactions, expressed as areas, we must always remember that to be comparable as magnitudes the satisfactions must be estimated by the same person. With these reservations we may now proceed to the diagrammatic representation of the estimates dealt with in the second chapter of Book I. and generally to the interpretation of curves of total and marginal satisfaction.

We may take (arbitrarily) a small square on the ruled paper of Fig. 9 to represent one-quarter of the satisfaction anticipated from the expenditure of a farthing. Then four squares will represent the satisfaction corresponding to a farthing, sixteen squares that corresponding to a penny, and $12 \times 16 = 192$ that corresponding to a shilling. Any rectangular or curvilinear area, irrespective of its shape, if equal to 192 small squares would then represent this shilling volume of satisfaction. It might, for instance, be a rectangle with a base of 1 and an altitude of 192, or one with a base of 16 and an altitude of 12.

Taking a side of a small square as our linear unit, let us

now agree that the unit length (not area) measured along any base line shall represent a periodic (monthly or as otherwise defined) supply of one ounce of tea, and a base of 16 such units a supply of one pound. We can now represent diagrammatically any of the data as to tea which we assumed in Book I. Chapter II. For instance, the fourth pound was expected to yield a satisfaction equal to the significance of 8s. in any other application. This would be represented by an area of 8×19^2 ; and as we have agreed that a basis of 16 shall represent a pound, a rectangle of base 16 and altitude $8 \times 12 (= 96)$ will be the proper representation of the satisfaction anticipated from the consumption of the fourth pound per month (Fig. 9 (a)). But of this fourth pound we saw that the first half was estimated at 4s. $5\frac{1}{4}$ d. and the second at 3s. $6\frac{3}{4}$ d. These values would be represented respectively by rectangular areas containing 852 and 684 small squares, and since the basis of each would, by our convention, be 8 (corresponding to $\frac{1}{2}$ lb.), their altitudes would be respectively $106\frac{1}{2}$ and $85\frac{1}{2}$ (Fig. 9 (b)). We can now interpret units of altitude. They will not signify positive quantities, as the units of the base do, but penny rates of satisfaction per pound of the commodity, or halfpenny rates of satisfaction per half-pound, and so forth.

Now, taking *ad* in Fig. 9 (b) at an altitude of 96 as in Fig. 9 (a), it is obvious that the rectangle *ab*, which is added to the original rectangle at the left, is equal to *cd* subtracted from it at the right, since the total area of the two differentiated rectangles is to be exactly equal to that of the integral rectangle that represents the satisfaction yielded by the whole pound; and we may suppose that this differentiation between half-pounds, quarter-pounds (or any other fractions, for it is not necessary to proceed by bisection of a pound rather than trisection, for instance), may be carried as far as we choose. The area of any succession of differentiated rectangles will always remain equal to that of the integral areas that present them collectively as a single magnitude. In Fig. 10 let us carry out this process to different degrees of advancement for the different pounds; and let us draw a curve such that in the case of the small and the large rectangles alike it always adds on an area to the left equal

to that which it cuts off to the right, so that for any base the area bounded above by the curve shall be exactly equal to the rectangle standing on the same base. Such a curve may be regarded as integrating any number of contiguous rectangles which we choose to take in succession. That is to

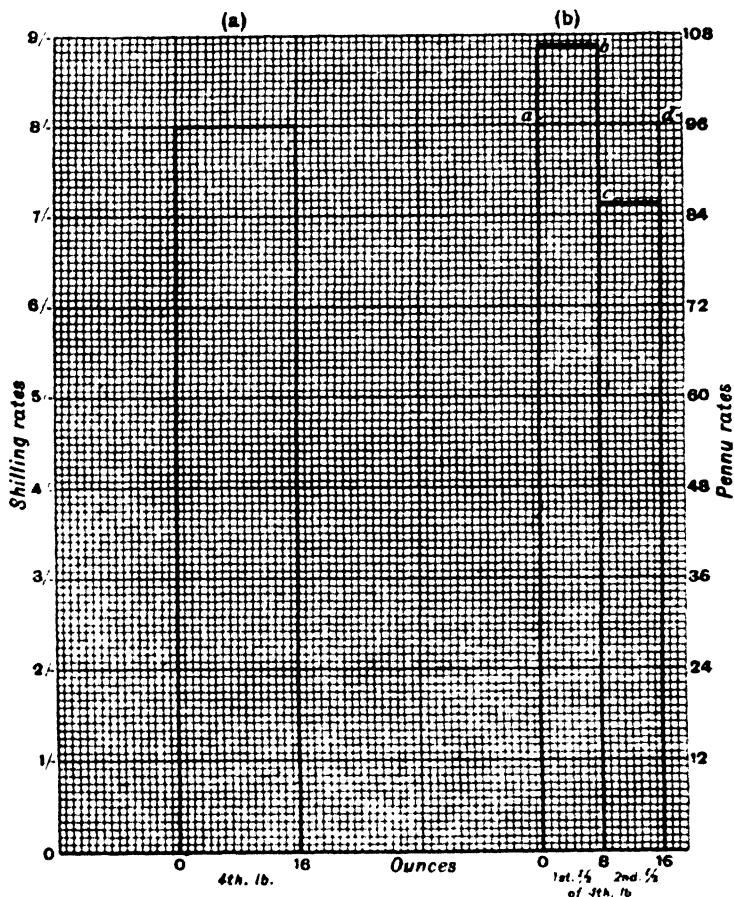


FIG. 9.

say, the area intercepted by the curve above any line measured along OX will be exactly identical with the area contained in the whole series of rectangles standing upon the same base.

This is a curve of total satisfaction, and its meaning is now obvious. We have seen that ideally, and in the limit, the significance of any commodity is a magnitude continuously

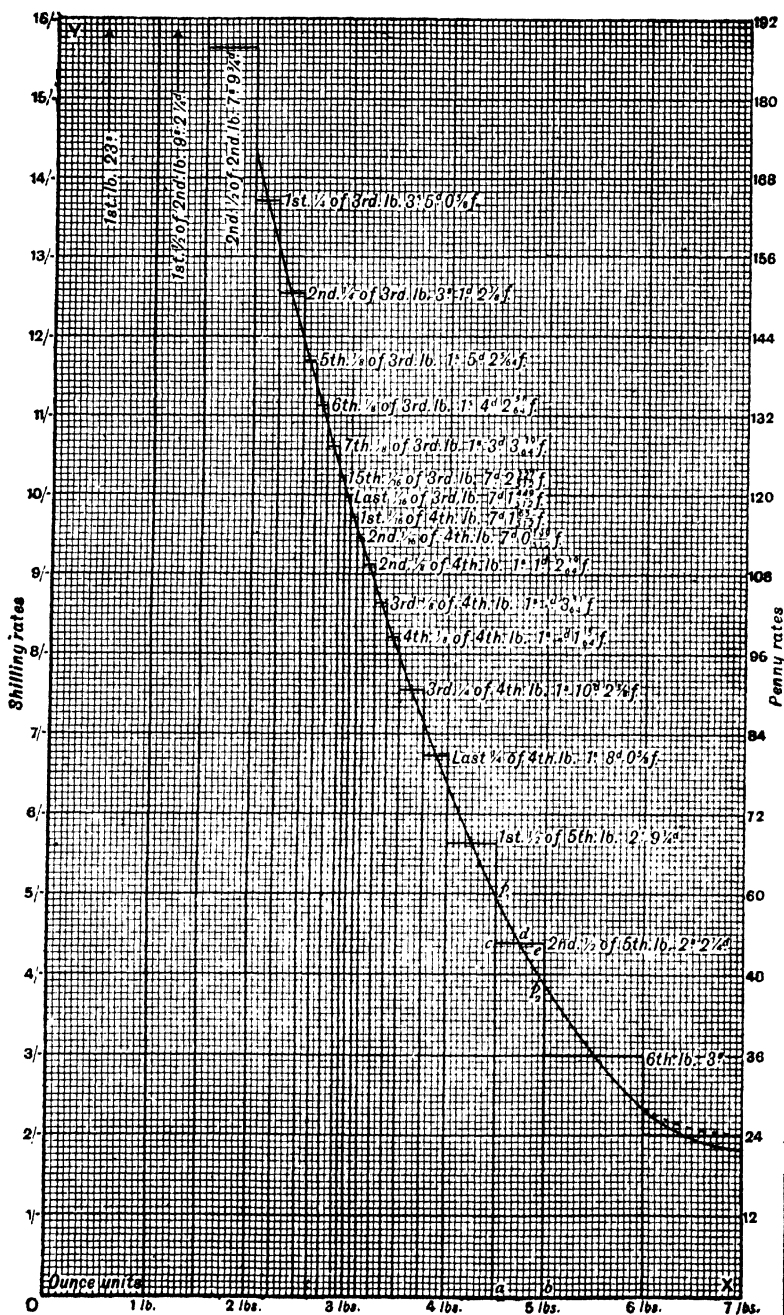


FIG. 10.

changing as we recede from the origin, so that, however small the increment we are considering, the change cannot be regarded as suspended during the progress of its consumption. The whole process, then, ideally considered, is properly represented not by a series of steps or discrete areas, however small, but by a curve-bounded space. Such a curve, could we obtain it, would give us at a single view the whole infinity of facts to be registered. If we take any portion of the weekly, monthly, or other periodic supply of a given commodity (whatever our conventional units may be), *e.g.* the third unit, or the quarter of a unit between $7\frac{1}{3}$ and $7\frac{7}{2}$, or generally the portion represented by the line ab on the axis of X (Fig. 11), then the curve is constructed so as to bound an area, ap_1p_2b , exactly representing the satisfaction anticipated from the consumption of the portion of the commodity represented by ab . And note that whereas in Book I. Chapter II. we directly assumed data as to pounds and binal fractions of pounds only, a curve assumes that we have all conceivable data, and can begin and end anywhere we like.

This continuity and entire accuracy of data is, of course, purely ideal. We may conceive approximations to it, but to imagine that any one could distinguish between the rate at which tea was ministering to his satisfaction at the beginning and at the close of his consumption, say, of the 7.9432th pound, and could express this difference in fractions of a shilling-per-pound rate, is an absurdity. Indeed the reader who has some tincture of mathematical culture will perceive that even an underlying assumption of commensurability between the satisfactions accruing from successive conventional units of the commodity and those represented by the conventional units of the currency is inconsistent with ideal accuracy. These reflections reveal at once the great convenience and the ingrained artificiality of the method of representing economic quantities by curves. The very nature of a curve is incompatible with the nature of the phenomena we are investigating; but it is of high value as an ideal simplification, and as a means of mentally arresting phenomena, which in their actual existence are unmanageably complex and fluctuating. If we professed in our diagrams to present possible or actual facts, we should have to undertake the hopeless task of

determining in each case what degree of accuracy might reasonably be assumed; whereas by frankly presenting the unattainable limit in every case we declare at once the ideal nature of our hypothesis and of our representation of it.

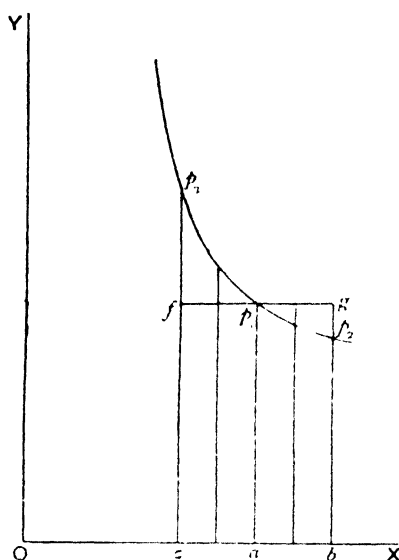


FIG. 11.

This being understood, the reader will have no difficulty (if he turns back to our investigations as to "limiting rates"

on pages 60 *sqq.*) in recognising the height ap_1 (if he turns back to our investigations as to "limiting rates" on pages 60 *sqq.*) in recognising the height ap_1 of the curve above any point a as the graphic representation of the limiting rate of significance of marginal significance. (in whatever unit measured) of increments or decrements of the commodity taken from the point a . For on considering the errors (p_3fp_1 and p_1gp_2 respectively) that would be involved in treating the areas above ab and ac as equal to each other and to the rectangle on base ab (or ac) and with altitude ap_1 , we shall find that they become smaller not only absolutely but proportionally to the areas themselves as we make the increments ab and ac smaller; and this without limit. For if we halve the lengths ab and ac and erect perpendiculars on them and then compare the rectangles on these bases, and with altitude ap_1 , with the areas above the bases bounded by the curve, we shall see that

the error involved in treating them as equal is in each case less than half of the corresponding error for the wider basis. The proportion of error, therefore, decreases, without limit, as smaller bases are taken. Thus the height ap_1 represents a rate of satisfaction per unit to which no increment or decrement taken from the point a ever conforms, as a whole, but which always lies between the rates proper to any given increment and to the corresponding equal decrement, and to which those rates approximate without limit as they decrease in magnitude. The units on OY , therefore, measure limiting rates of the significance of units of the commodity (per unit of time) as the increments are taken smaller. Or, in abbreviated terminology, the ordinates represent the marginal significance of the commodity for any given supply. So, too, in Fig. 10 the areas p_1cd and p_2ed respectively will be not only smaller, but smaller in proportion to the rectangles da and db as c or e approaches a .

We have now a provisional conception of what a curve of marginal significance would mean if we had it, and we may go on to the examination of the bearing upon the determination of the form of such a curve of any data we may suppose ourselves actually to command. Construction
of a curve
of total
satisfaction. Let us rule our paper, as in Fig. 10, so as to mark rectangles of base 16 and altitude 12. Returning to our example of tea, we may retain the significance of all our units, and for convenience may register successive pounds (each pound being 16 ounce-units) of supply along OX , and successive shilling-per-pound rates of significance (each being 12 times a penny-per-pound rate) along OY . Each large rectangle, containing 192 small squares, will indicate, as before, the area of satisfaction represented by a shilling.

It is obvious, to begin with, that any datum we may be able to obtain will give us some information as to the course of the curve. If we know, for instance, that the fourth pound of tea yields an area of satisfaction valued at 8s., we shall know that the curve must be such that the area ap_1p_2b equals the area ac , and the area p_1df equals the area $fc p_2$. (We shall express compliance with this condition by saying that the curve "satisfies the datum" of the area ac .) But there is an infinite number of curves that would fulfil this condition.

Some of them might bisect dc , and others might cut it at points indefinitely near to d or c , and they might intersect the verticals from a and b at any variety of points. But if we have the additional data that the first half of the fourth

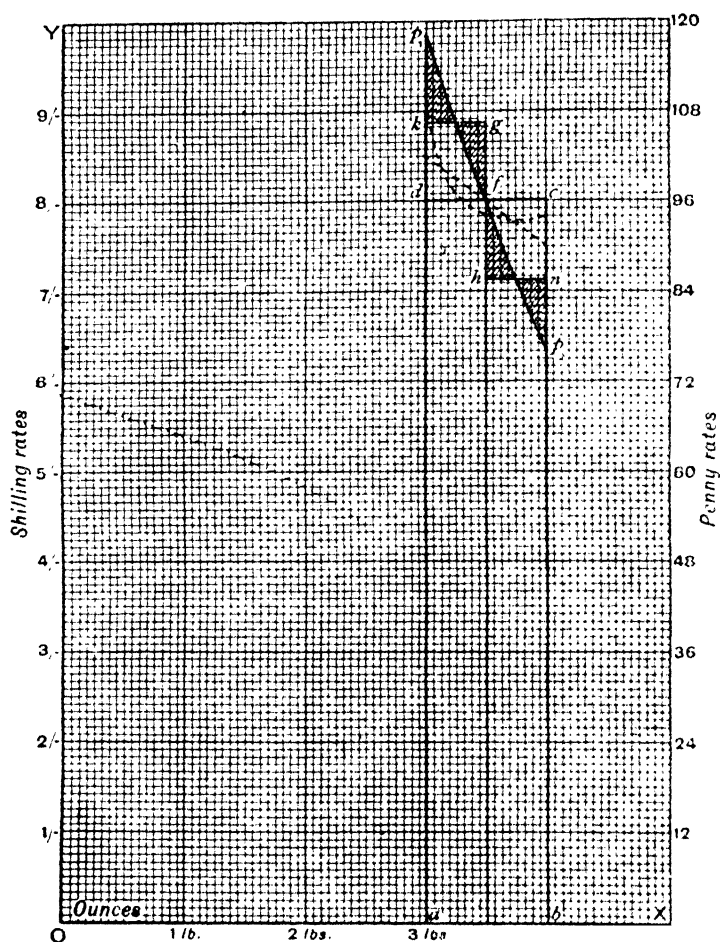


FIG. 12.

pound corresponds to the area ag , and the second to the area bh , many of these possibilities will be excluded, for the area which the curve adds to ag must equal the area it cuts off from it; and the same must hold for bh . The course of the curve, therefore, will be more closely determined by the two

rectangles ag and bh than by the one rectangle ac , which is equal to their sum. In our original hypothesis we supposed the estimates of successive pounds of tea to reveal an easily detected law which enabled us at once to calculate any smaller areas we liked to choose. This formula would absolutely determine the form of the curve, and tracing it would only be a matter of calculation. But if we assume no such property, and imagine each datum to stand alone and not to involve any derivative data (assuming only the general property of continuous decline, after a certain point, which we may take as fixed by the nature of our inquiry), then it is clear that the minuter the increments for which we can obtain estimates, the more closely can we determine the course of the curve. For instance, we have set out on Fig. 10 (page 444) a series of data as to pounds, half-pounds, etc., and we see that, *so far as they shew* (that is to say, apart from our knowledge that our formula would enable us to split up the larger rectangles as finely as we choose), there would be room to suppose that the curve undulated with considerable violence over the portion corresponding to the increment from 4 lbs. to 7 lbs., but that our data enable us to assert a more regular course for the portion corresponding to the increment from 2 lbs. 12 oz. to 3 lbs. 4 oz. Seeing then that if we have given any two contiguous rectangles of satisfaction, $akym$ and $mhnab$, the curve must always pass between the points g and h , it follows that if we could determine the areas corresponding to indefinitely small increments we could determine the position of the curve at any part of its course within indefinitely narrow limits, for just as we determine a point absolutely if we can determine any position we choose of points, that approach each other without limit, between which it lies,¹ so we can determine a curve absolutely if we can determine, as closely as we like, two mutually approximating points between which the value of y , corresponding to any given value of x , lies.

But here it will be well, for our security, to establish the fact that whereas (as we have just seen) a curve may satisfy the datum of a certain area, but may fail to satisfy the data of two smaller areas into which it can be broken up, it is

¹ See page 60.

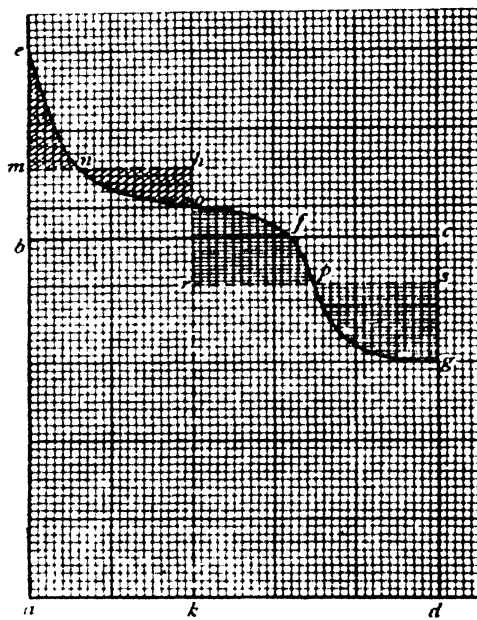
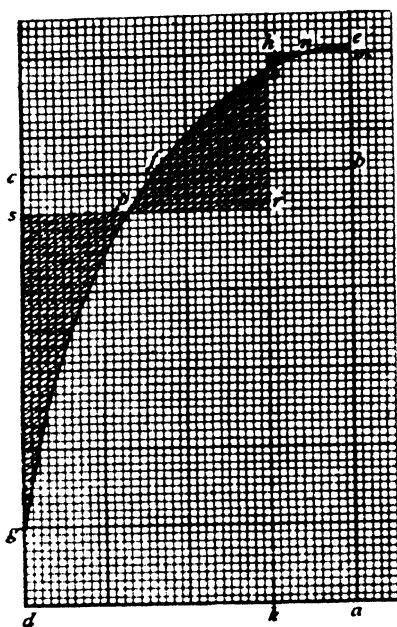
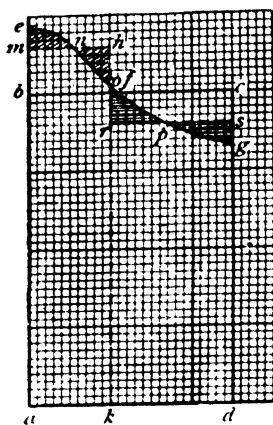


FIG. 13.

not possible for it to satisfy the data of two adjacent areas, severally, without also satisfying the data of the total area which is their sum. The general proof of this proposition, to which we will now proceed, applies to all the different forms of curve shewn in Fig. 13.

We start with the two rectangles ah and ks and construct a curve, $enofpg$, such that it adds and subtracts equally from each of the two rectangles. The equal areas we mark by oblique or horizontal lines respectively. There are, of course, an indefinite number of such curves; but if we construct an integrating rectangle, ac , by drawing a line, bc , that makes the rectangles bh and rc equal, the area which the curve $enofpg$ cuts off from the rectangle ac will be equal to that which it adds to it—that is to say, the area ebf will equal the area gcf . Since we have $emn = nho$ we may substitute the latter for the former, and we shall have $ebf = bmhof$. Again, since we have $bh = rc$, we can obtain by substitution $bmhof = scfor$. And since we have $rop = psq$, we can again obtain by substitution $scfor = gcf$. Therefore we shall have $ebf = gcf$. Q.E.D.¹

Thus, if we have any series of rectangles arranged as in Fig. 10, on bases measured continuously along OX , a curve which adds to and cuts off equally from any contiguous pair of these rectangles, severally, will have the same property with respect to the integrating rectangle that is equal to their sum. The rectangle so obtained may then be substituted for the two rectangles of which it is the sum, and we may again integrate it with another rectangle, still relying on the same result, so that the curve will always add and subtract equally from the area of the integrating rectangle that sums any number of contiguous areas with the data of which the curve complies severally.

It is evident, therefore, that since we can always rely on the curve's retaining its fundamental property when we add together the data on which we build it, but never when we subdivide them, the accuracy with which we can determine it will depend on the accuracy and the fineness of the data on which we can construct it.

¹ This analytical proof is, strictly speaking, unnecessary; for since we have $ah = acok$ and $ks = kqnd$, we have also $acqd = ah + ks = ac$; and this involves the equality of ebf and gcf . But the proof by substitutions may probably be found the more enlightening.

To what degree of approximation, then, can we hope actually to determine such a curve? Or, rather (since the question so put hardly admits of a definite answer), what are the principles which will determine the degree of approximation to an ideal curve that may be realised in any particular case? In the first place, let us consider the question of accuracy.

Conditions
that determine
degrees of
precision in
determining
such curves.

In the case of the tea curve, for instance, we have to ask what will determine or influence the limits within which we can reasonably suppose our housekeeper's estimates to be exact. But on the very threshold of this inquiry we are met by a grave difficulty. What do we mean by accuracy of estimate? If we are speaking of the estimate a man forms of the length of a stick, for example, or the height of a top-hat, we are speaking of something which can be tested by actual measurement. Thus if we say that a man can be trusted to judge a yard to within a quarter of an inch, we mean that if he declares such and such a thing to be exactly a yard long, or undertakes to measure with his eye a yard length from any given point, we shall find on testing it by standard measure that what he pronounces to be a yard will not be less than $35\frac{3}{4}$ inches, nor more than $36\frac{1}{4}$ inches. But what could we mean by saying, for instance, that you could rely on a housekeeper's estimates of the significance to her of such and such an amount of tea, under such and such circumstances, to a farthing? She is making an estimate, and if that *is* her estimate, what is the meaning of calling it accurate or inaccurate? Even if you try to bring it to the test of experience, and ask her afterwards whether her estimate is justified by the result, she can only tell you that it has or has not procured a satisfaction equal to what she now supposes she could have got by the sum she mentioned, if she had applied it otherwise; and this is itself an estimate. Though her estimates, therefore, are based on experience, and are checked and modified by it, yet no objective standard of experience can be kept for reference, or can be applied objectively as a check, like the standard yard.

Apparently, therefore, what we should mean, in the first instance, by saying that a housewife's estimates, under certain conditions, will be reliable to a farthing, would be something like this:—If we are dealing with estimates, as such (and not

with the experiences which might or might not correspond to them if the experiment were made), we shall find that they may be made in various ways. We might ask a housekeeper to say how much another half-pound of tea would be worth to her if she already had $2\frac{1}{2}$ pounds, and then some time afterwards, when she had not that question and answer in her mind, we might ask her what half-a-pound would be worth if she had 3 pounds. Then again we might divide the amount into other fractions of a pound, thirds or fifths, and begin at some other base than $2\frac{1}{2}$ pounds, but include the former area in our new inquiries. And finally, we might ask how much a whole pound would be worth if she already had $2\frac{1}{2}$ pounds. Now if she answered all these questions independently, giving every answer on the strength of a direct estimate, without mental reference to previous answers, and if the answers when compared never revealed inconsistencies of more than a farthing in the pound, and if similar tests produced similar results wherever applied, we could say with confidence that her estimates were not mere guesses or random selections of prices or quantities on which her mind was accustomed to rest, but were direct and genuine quantitative estimates, accurate as estimates, and therefore consistent, to within a farthing a pound. Another test would be to present the same question at different times in such different lights or connections as to suggest different answers, and see whether such suggestions or associations influenced the answer.

This must be the primary meaning of accuracy and reliability of estimates as such. But behind this we may think of the correctness of the estimates as attempts to realise hypothetical experiences. We may have a clear and consistent idea of the value we should attach to such and such a supply of a commodity if we already commanded just so much of it and no more, and it may be impossible to shake that estimate by the most skilful cross-examination; but yet if the experience comes we may find that we had formed a very erroneous conception of it, and our estimates may be very different now from what they were when the experience was only hypothetical. Thus remoteness of the supposed case from experience may either affect the precision of our estimate as such, or it may make our estimate

Meaning of
accuracy in
estimates.

now (whether precise or vague) unreliable as a forecast of what our estimate would really be under other circumstances. These two things must always be distinguished in our minds, though it may not always be necessary to insist on the distinction in any particular context.

But yet again. It is impossible to banish the idea that as well as more or less imperfect estimates there are certain definite and ultimate facts to be estimated, and that faults or errors of estimate do not affect these ultimate facts. How can we get at precise conceptions in this matter? Clearly we are still dealing with subjective experiences and not with external magnitudes. But just as we know that many impressions are received by the eye but not consciously registered by the mind, so there may be many sensations and experiences that actually go to making us happy or strong or the reverse, but of which we are not conscious as causes, or which are in themselves so slight that we have not learned to pay attention to them. An ideally perfect estimate would identify every cause and register every effect, and would actually assign to all experiences the values they *would* have for us if we distinctly realised them. We can reach no conception more nearly approaching objectivity than this.

Returning now to our actual estimates as such, we may go on to examine some of the influences which make a greater or lesser degree of accuracy, in the sense of precision and consistency, possible in any given case. But it will be well at this point to develop a distinction that has already been made, though not emphasised.¹

Conditions that affect precision and minuteness of estimates. Accuracy is not the only valuable quality in our data, for we have seen that the curve which satisfies the minuter will always satisfy the broader data, and the minuter data determine the curve more closely than the broader. Minuter data of a certain relative inaccuracy might therefore determine the course of the curve more closely than the broader data of relatively greater accuracy. In Fig. 10, for example, we might suppose that the area of satisfaction corresponding to the sixth pound was given with great accuracy, but if we had no minuter data the curve might, for anything we should know, undulate in an indefinite number of ways, within wide

¹ At the bottom of page 451.

limits, over that portion of its course. We should have one accurate datum, but the course of the curve would be indeterminate; whereas we might suppose a considerably higher degree of proportional inaccuracy in our data at and about the end of the third pound, and yet be more certain that we had determined the course of the curve about that point within narrow limits. The relatively inaccurate data, because narrower, would exclude many possibilities which a more accurate datum, if broader, might admit. And, as we shall see, it may very well happen that the broader data are, as a matter of fact, proportionally more accurate than the narrower. In such a case the narrower data may be of service to us in determining the general course of the curve within the limits of the broader data, but owing to their relative inaccuracy in detail their summation might give results incompatible with the broader data, and in such cases we should be guided by them only in such a general way as is consistent with compliance with the less determinate but at the same time more accurate conditions implied in the broader data.

With this proviso we will proceed with our examination of the conditions favourable to precision and consistency of estimate. Some general remarks on precision in estimating objectively measurable magnitudes may precede our examination of the more evasive estimate of satisfactions as magnitudes.

We must not blink the difficulty and complication of this problem, or the fact that any general principles we can lay down will be subject to every kind of disturbance from the personal idiosyncrasies or the special experiences of the individual who makes the estimates. It will, however, be admitted that in estimating quantities of any kind, a given individual will have a range, or theoretically a point, of maximum accuracy. Take an observer whose experience, professional or other, gives him no particular guidance in the matter, and present him successively with two pieces of wire, one an inch and the other an inch and a half long; then, successively, with diagrams shewing spaces of $\frac{1}{3}\frac{1}{2}$ in. and $\frac{3}{4}$ in. respectively, intercepted between fine lines. Then take him to a place from which he has a variety of views, and under conditions identical as to distance, angle of observation, and

so forth, ask him to notice the distance between the trunk of a tree and a boulder (known by you to be 1000 yards), and subsequently the distance between the edge of a tarn and the edge of a snow patch (which is 1500 yards). In each case ask him what proportion the first length in each pair bears to the second. You will probably expect a more accurate proportional estimate in the case of the inch and the inch and a half than in either of the other cases. Perhaps there will be some other length which he will be able to estimate more accurately still, but there will be some point, between the thirty-second of an inch and 1000 yards, in the neighbourhood of which his estimate will reach the maximum of accuracy. And as he recedes from this in either direction his estimate will become less reliable. It does not follow, however, in individual cases, that this departure from accuracy will be regular and continuous. There may be certain definite magnitudes which, for one reason or another, the individual has been accustomed to measure with unusual accuracy, and these may be irregularly distributed. Thus, if we take a carpenter who is also a professional cricketer, and who, when a boy, sometimes ran along a mile of road keeping pace with a stage-coach, and if we submit to him pairs of lengths which are really the same fractions of each other in every case, and not very remote from equality (say that one is nine-tenths of the other), probably if their mean is a foot he will estimate them with greater proportional accuracy than if their mean is 9 yards. But again he will measure them with greater accuracy if their mean is the 22 yards of a cricket pitch than if their mean is 9 yards; with less accuracy if their mean is 1000 yards than if their mean is 22 yards; but with greater accuracy again if their mean is a mile than if their mean is 1000 yards. Thus, the general principle that there is a certain magnitude in the neighbourhood of which estimates reach a maximum of accuracy from which they depart in either direction, may be qualified by any vivid experience or frequent practice which may have cultivated particularly accurate observations of certain lengths. And whatever the points of maximum accuracy may be the man will attempt to reduce his problems, when possible, to terms of the lengths he can best judge. Thus if a length is unmanageable he will try to

divide it into halves, thirds, or quarters, or to multiply it by two or one and a half, and see whether these fractions give him lengths that he can judge immediately with some confidence and from which he can then calculate the others. The boy who, when asked how he would estimate the distance of the sun from the earth, answered, "Guess a quarter and multiply by four," had a confused sense of a sound method in his mind, though he was not fortunate in his application of it.

Now in the case of our tea curve all these complications are present, and certain others as well. The ultimate quantities to be estimated and compared, here as elsewhere in the administration of resources, are not tea-leaves and pence, but quantities of satisfaction; and yet the housewife is never accustomed to think of these as quantities at all. She thinks in pounds and ounces of tea, and in shillings and pence of money, but the half-unconscious and wholly unanalysed processes which emerge into conscious deliberations under these denominations of ounces and pence really concern lots of satisfaction. Hence a divergence between the points on which her deliberations crystallise themselves in her own consciousness and those on which they actually depend.

It is not difficult to see why this is so. In order to estimate tea with reference to other commodities we must express its value in terms of money, as the common measure between all the commodities in question; and we shall estimate it in the quantities in which we are accustomed to buy it. But our direct experience of its value is based on much smaller units, for while we pay for tea by the pound we consume it by the cupful. If a man drinks two cups of tea of a certain average strength every day for breakfast, his estimate of the value of a pound of tea must be arrived at by considering it as supplying, say, sixty-four breakfasts, and the marginal value of a quarter-pound by considering the significance of substituting a cup and a half for two cups at these sixty-four breakfasts. The enjoyment of tea at one breakfast is the quantity of satisfaction he really estimates, but in order to bring it into correspondence with his problems of expenditure he must reduce it to the terms in which he actually deals

Relation
of psychic
quantities to
units of the
currency.

in it. If we express our estimate of one sixty-fourth of a pound of tea in terms of money we fall into manifest absurdity. For money is an instrument of practical exchange, and since we cannot give effect to these minute estimates of a fraction of a farthing in any actual transaction, this method of expression loses all its value. Hence the sense of intolerable unreality in our previous working out of the tea problem (pages 44-63). As we narrowed the areas of our estimates and so brought ourselves nearer to the actual basis of realisable experience we continued to express those estimates under a denomination that was becoming more and more hopelessly inappropriate and unconvincing.

Thus the point at which we deliberate as to alternative expenditures of money is likely to be remote from that at which our experience gives us the most direct and vivid sense of the immediate value of a commodity. In a word, to compare one *expenditure* with another we have to recede indefinitely from the points at which we can best compare one *experience* with another. Commodities are not practically exchanged with each other, or obtainable as alternatives, in the quantities in which the experiences they provoke are most directly comparable with each other. And as we are more accustomed to deliberate consciously as to expenditure than as to satisfaction (though our whole expenditure is ultimately regulated with a view to satisfaction), a difficulty inevitably arises. The careful administrator does occasionally revert consciously to the primary and ultimate basis. She may from time to time calculate, for instance, how many rice puddings can be made out of a pound of rice, or how many breakfasts a pound of tea will provide, in order to establish a kind of bridge along which she may pass either way from the quantities in which she buys commodities to the quantities in which she experiences their services. She sometimes travels from her expenditure per pound or per annum to her satisfaction per quarter-ounce or per diem, in order to base herself upon experience, and she sometimes calculates how much a saving too minute to be estimated in coin of the realm day by day would amount to in a month or a year, in order that she may bring one set of experiences into terms under which it may be compared with another and alternative set.

As we are now to deal with the ultimate limits of accuracy in the construction of a curve, it is obvious that we are concerned not directly with shillings and pence per pound, but with the estimates of satisfactions ^{Ultimate} per cup, and so forth, as quantities. Obviously it ^{psychic basis} is with these that the housewife must ultimately wrestle. For instance, if an economy is to be effected she may have tea at fewer meals, never supplying it at certain times of day unless it is expressly asked for, or in the last resort saying that it cannot be had; or instead of this she may make it weaker, or she may practically limit the amount of the infusion at each meal while not limiting the amount of hot water that passes through the pot, or she may look for a cheaper tea, or (*horresco referens*) one that will not be so popular in her household. She may or may not be subject to such more or less unsympathetic pressure from her family as is implied in some of the foregoing suppositions, but in any case she is dealing with certain alternatives, and in considering them she is estimating and comparing volumes or areas of satisfaction, and it is a reference to these that underlies her estimates in money of the marginal value of an ounce of tea, and determines at what point of pressure she will buy more or less of any given quality at any given price.

It is therefore here that we must apply the principle of the magnitude that is estimated with greatest proportional accuracy; for there may be some one or more of the satisfactions she habitually considers which, as magnitudes, are realised with especial distinctness and vividness, and to which others are consciously or unconsciously referred as to a kind of standard. Suppose, for example, there is one member of the household whose wants, for any reason, good or bad, the housekeeper considers it specially important to satisfy, and whom she occasionally disappoints, as to quality or quantity, in the matter of tea. The significance of this occasional contretemps may well constitute the actual unit of greatest proportional accuracy of estimate, and it may be by unconscious reference to it that the housewife can determine most accurately the relative values of all the alternative refusals, indulgences, evasions, devices, and pecuniary expenditures, with which she is concerned in the matter. Here, as in the case of the

carpenter, there may be other points impressed by other experiences that give an exceptional degree of firmness to estimates of certain other quantities ; but, neglecting this consideration, we may follow up the special clue we have grasped.

Note that our housekeeper will probably never deliberately incur or inflict the specific privation we are considering, merely in order to economise the tea needed to avert it. It will occur through some inadvertency or miscalculation, and it will be the delay, or trouble, or want of courtesy to a guest, or incidental (as distinct from primary) waste, that would be involved in correcting the error that will determine her to accept the result. But when the housekeeper is asked to make a number of hypothetical estimates as to what successive increments of the supply would be worth to her, and comes to think of a contraction of supply great enough to make this specific privation normal and permanent instead of occasional and accidental, she finds she has a very clear conception of that particular "lot" of satisfaction, that she has been accustomed to translate it into a great variety of equivalents, and that she has from time to time defined it pretty closely as worth just so much of certain other things, but not even a little more. She can now translate it, by a deliberate calculation, into so much tea per month, and can estimate it with some precision at its money value. This may form a kind of standard unit of reference, and may be the magnitude she is capable of estimating with the highest degree of proportional accuracy and precision. The area thus determined will be that of the elements out of which our curve can be constructed with greatest accuracy. For in considering the value of other increments nearer to the margin or further from it, our housewife (we are supposing) will find it easiest to make accurate estimates of areas of satisfaction of this particular magnitude ; and she will find, of course, that if she has to think of herself as compelled by the further contractions of her supply to cut deeper back into the satisfactions of her household than she has ever actually done, she will realise that a smaller amount of tea, at the higher significance so reached, would yield the standard unit of satisfaction, and that in like manner at a more advanced point it would require a correspondingly larger amount to secure it. Geometrically the standard area will

stand on a narrower basis as we approach the origin, and on a broader one as we recede from it.

Thus, subject to all the qualifications hinted at or developed, we may suppose that the ultimate elements out of which data for the curve would be obtained with the greatest proportional accuracy would consist in estimated satisfactions of a magnitude about equivalent to that of the satisfaction relinquished on the occasions of disappointment that have impressed themselves most vividly on the housewife's mind. They would be represented on our diagram (when reduced to the terms of a month's supply, and expressed in shilling and penny rates per pound) by a series of rectangles of uniform area standing on progressively larger bases as we recede from the origin.

Elements from which the curve is constructed.

Now seeing that every day the housekeeper deals with the whole supply for the day, and has the opportunity of experiencing or observing the actual service rendered by every increment from the initial to the final one, we might be tempted to think that she could base her whole conjectural construction of the curve from the origin to the margin upon direct experience. But this is not so. We have seen that recurrently satisfied wants never take us back to the real initial significance of the things that satisfy them.¹ If our supplies were very much contracted (even apart from any reaction upon the organism that might ultimately take place) we should gain experience of significances that had evaded us before; for the want which to-day's first increment supplies is a different want according to the point up to which our want was satisfied yesterday. And as soon as we begin to contract or increase our supply at all this process sets in, though its effect at first may be hardly perceptible, and it may only become pronounced as we recede considerably from our present margin. Thus an additional element of uncertainty enters into all estimates far behind or far in advance of the present margin, and our ideal equal areas will become correspondingly more speculative. This speculative element may reveal itself consciously in a refusal to make equally precise estimates, or unconsciously in an inability to make equally consistent ones, as we recede from the actual

Range of experience

¹ Cf. page 426.

margin. Past experiences, vividly remembered, may establish at irregular intervals other bases of comparatively direct and immediate estimates; or critical points may so appeal to the imagination as to give a firm but illusory precision to speculative estimates; or some changed unit of maximum accuracy may assert itself in certain regions of the curve; and throughout we must distinguish between precision and consistency in the sense explained above, and approximation to the estimates which would be formed under the pressure of immediate experience should it ever be realised.

When formed, our curve, such as it is, will be an estimate, or a register, more or less reliable, both of the total significance to be derived from the consumption of any given quantity of the commodity, and of its marginal significance at any point.¹

Before leaving this branch of the subject we may note that if we asked for estimates of the significance of a series of objectively equal increments of the commodity we should have a series of rectangles, not of equal area but on equal bases, from which to construct our curve; and we may ask what conditions would influence the delicacy and accuracy of our estimates of the difference of area between them. Two considerations are relevant here. In the first place, the same magnitude is less easily perceived and estimated as part of a larger than as part of a smaller whole. The difference of an inch is more conspicuous in the length of two men's noses than in their heights. Small differences will therefore be less delicately noted when the areas are large than when they are small, and therefore a given difference between two contiguous rectangles might escape detection near the origin but might be distinctly felt farther from it. But in the second place, our whole investigation has shewn us that the significance of successive increments of the commodity changes more rapidly in some regions than in others. Between two successive rectangles on equal bases, therefore, we shall sometimes have greater differences and sometimes have keener powers of observation. The first condition is indicated by a rapidly falling curve, and the second by a higher positive altitude of the curve. In our example of the tea, and in Fig. 14, *a*, these two conditions tend to counteract each other; for as the

¹ But see below, pages 467-499.

differences themselves decrease, our power of perceiving them increases. But in Fig. 14, *b*, they reinforce each other. As the differences themselves become greater our power of observing them also becomes more acute.

Enough has now been said to shew in the first place how extremely precarious any actual evaluation of a curve of total significance of any commodity must necessarily be, but also, in the second place, that this value, which it is so difficult to estimate, is actually a definite and a highly significant quantity.

The area bounded by the curve represents what the older economists called the "value in use" of the commodity, that

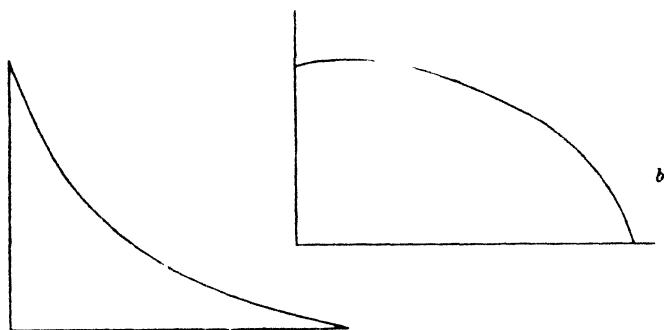


FIG. 14.

is to say, the total satisfaction or advantage derived from its enjoyment; and the height of the curve above any point on the abscissa represents its marginal significance, which, in the case of exchangeable things, will always tend to be brought into coincidence with its "value in exchange." And note that if our expenditure is wise a decline in marginal significance due to an increased supply will always coincide with an increased volume of satisfaction. A reduction in the "exchange value" of any commodity, taken in itself, should always result in its increased "value in use" to us.¹

We have now sufficiently examined the general meaning of a curve of total significance or satisfaction, and we have seen the very precarious nature of the data upon which any attempt actually to evaluate the total significance of a

¹ Cf. pages 45-47.

commodity must depend. But we have still to take note of certain points, a neglect of which might lead to erroneous or inaccurate thought.

It will be understood that a curve proves nothing whatever as to the facts from which we start. It is merely an idealised picture of facts and their implications. It

Form of the
supposed
tea curve.

may therefore enable us to understand the full meaning of any set of supposed facts, but it cannot establish them. At most it can only shew us the relations in which certain facts, if they exist, stand to each other. But by doing this it may bring out implications involved in our data that we had not fully realised, and this may throw back light on the validity of the data themselves. For instance, a glance at the tea curve at once suggests that it will not decline any further after the point to which we have carried it; and as there is no reason why the law of declining significance should become invalid after seven pounds, we begin to suspect our data of being in some way self-contradictory or impossible. And this is really the case. We supposed our original data as to the values of successive pounds of tea to conform to a perfectly rigid and easily discernible algebraical law. But this is strictly impossible. In the first place, it is impossible that the estimates should be mathematically accurate at all. That is to say, it is impossible that an infinitesimal change in the quantity of the commodity could be actually and directly appreciated, and its significance registered in terms of money. But if we are dealing only with approximations it may possibly happen that the more or less loose estimates given may conform loosely to some simple algebraic formula. Since, however, an immense number of heterogeneous factors would enter into the composition of every region of the curve, some of them changing as it proceeds, we may be very sure that no simple algebraical formula would be able to represent them all even approximately, though it might approximately fit a certain portion of the curve. So if we had assumed this precise algebraical law as determining the whole curve, we should have assumed in the first place an impossible precision, and in the second place a highly improbable (and, as it turns out, impossible) simplicity and regularity. As a matter of fact it will be found that our original data themselves assumed



that after the sixth pound the law of the curve would change ; for the series 23s., 17s., 12s., 8s., 5s., 3s. would give as its next term 2s., and we have constructed the curve on this estimate. But this contradicts our original data, for we started with the supposition that at 2s. a pound the purchaser would take 7 lbs. ; and the figure makes it very clear that if the whole seventh pound is only worth 2s., then the first half-pound is worth more than a shilling, and the second half-pound worth less. The second half-pound therefore would not, on this supposition, be bought at all. Our curve would give about 6.42 lbs. as the ideal point at which the purchase would stop. So if we are to suppose that 7 lbs. would be bought at 2s. we must suppose the character of the curve to change after 6 lbs. It might take some such course as that indicated by the dotted line.

In very many cases a curve that approximated to a similar formula during a part of its course might reasonably be expected to change its character as it approached the origin ; for we have seen that at first a commodity may have increasing significance, and may only enter upon the period of declining significance "after a certain point."¹ In the case of tea, however, there is nothing palpably absurd in supposing our curve to follow approximately the formula we have assumed, at any rate up to a very close proximity to the origin. It is easy to imagine that as tea (or coffee) became dearer and dearer a careful housekeeper, whose family still retained a taste that they were less and less able to indulge, might limit the purchases more and more till at last it was only on occasions of special festivity that the precious infusion was consumed. When the price of £1 : 6 : 4d. a pound was reached, a quarter of a pound, or two ounces, might be bought for Christmas Day, and none at all at any other time. This consumption (four or two ounces a year) would be at the rate of one-third or one-sixth of an ounce per month, and would be represented on our figure by a point only one-third or one-sixth of the side of a small square from the origin. And if we had lowered the whole curve by, say, two of the large units on *Y* so that it intercepted the axis of *X* at a little under 6 lbs. 7 oz., the whole series of marginal values from the initial

¹ Cf. page 435.

increment to the one that completed the full satisfaction of the desire might, without palpable absurdity, have been supposed to be represented by this particular curve. As it is, it is clear that our original data involve the supposition that the law indicated by the successive steps in declining value from 1 lb. to 2 lbs., etc., up to 6 lbs., would not continue to hold for the decline from 6 lbs. to 7 lbs.

Even if we do not assume an algebraical formula for a curve, we can seldom use this diagrammatic method without expressing more and expressing it more precisely than we desire, and this constitutes a grave disadvantage in the use of curves for popular demonstrations. If, for example, we say that successive increments of a commodity will decline in significance after a certain point the statement remains general. But if we illustrate it by a curve, the "point after which" will be determined and the rate of decline at every point will be determined, and a general conception of the modes of variation will be suggested. And so the incautious student may be misled by the characteristics of the individual curve selected, and may fail to distinguish between them and those characteristics really involved in the data. The utmost caution is needed to prevent a curve from surreptitiously insinuating into our minds suppositions which are not included or involved in our data, but which we nevertheless receive into our conclusions. Nor is it beginners only that have fallen into this trap.¹ But this by the way.

We might now suppose that in such a diagram as Fig. 15, if properly constructed, we should have an ideal presentation of the amount of the commodity Ox that would be purchased by a certain individual at any given market price Oy ; of the total satisfaction Oy_0px that its consumption would afford; of the volume of other satisfactions Op sacrificed in the total sum paid for it; and of the surplus of satisfaction yy_0p which is secured over and above what is sacrificed. If this were so, then this last-named area would represent the advantage which the consumer derived from the existence of this particular market, and the volume of

Interpretation
of a curve
of total
satisfaction.
Instability of
the psychic
meaning of
the unit of
currency.

¹ Cf. pages 552 and 568 *sup.*

satisfaction of which he would be deprived if it closed or became inaccessible to him, all other things remaining equal.

These conclusions, however, are still subject to sundry modifications and qualifications which we must now examine.

In constructing our curve, we have used denominations of shillings and pence simply as measures of certain definite satisfactions, and we have tried to shew how, ideally, the area of total satisfaction corresponding to any given supply Ox of the commodity could be actually evaluated in these denominations. But on closer inspection we become aware of a disturbing instability and ambiguity in our unit when regarded as a psychological magnitude. We have often noted that 1s. has a different psychological significance to two

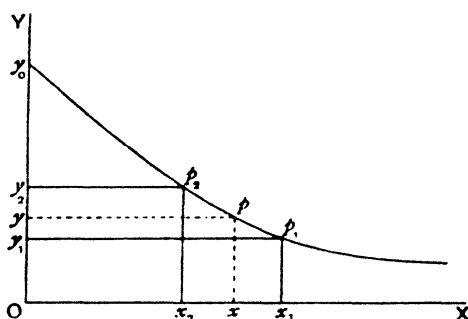


FIG. 15.

different men, and also to the same man if his income rises or falls. Theoretically, then, the marginal significance of a shilling will be affected by the sum the man has already paid to secure a certain satisfaction. We supposed, in our example of the tea, that the housekeeper gave us the outside value of the first pound of tea to her, and then *supposing herself to have paid that sum* for it went on to give us the outside value of a second pound, and so forth. If our Fig. 15 has been constructed on this system, then x_1p_1 will represent the marginal value of a commodity to a man, on the supposition that he has actually paid the money represented by the area $Oy_1p_1x_1$ for the quantity Ox_1 . But will Ox_1 represent the amount he would actually buy if the market price were Oy_1 ? Not unless the sum of money represented by the whole area $Oy_1p_1x_1$ is so insignificant a part of the man's total expenditure that it

makes no perceptible difference to the marginal significance of a penny whether the area $Oy_0p_1x_1$ or only the area Op_1 has been spent upon tea. If this is not so, then the fact that he can actually get Ox_1 for the expenditure of Op_1 will leave him better off than on our first supposition by the area $y_1y_0p_1$; and this being an appreciable sum it will enable him to get a little more of everything or anything (including the commodity under direct consideration) than he would have been able to do had he spent $y_1y_0p'_1$ (as well as Op_1) on the supply Ox_1 . A little more than Ox_1 may therefore be purchased. And again, since all the man's wants will be satisfied down to a lower point of urgency, the significance of what a penny will buy at this advanced margin is lowered. Thus the psychological significance of our unit will be smaller if the whole supply is purchased at the lower price than if the full sum represented by the mixtilinear area had been given for it. As we imagine Ox to advance or recede, the changing values of the total or the rectangular areas will react upon the psychological significance of the unit, and the difference between them will prevent the abscissa from accurately representing the amount that would be consumed at the price represented by the ordinate.

This is not a mere fanciful speculation. If a careful housekeeper were giving any such estimates as we have supposed, when she came to think of herself as paying 50s. or 60s. a month for tea instead of something like 14s., she might be perfectly conscious of the constraint she would feel in all branches of her household expenditure, and might realise that she was estimating the increments of tea in a unit of higher significance than that by which her actual expenditure is regulated.

The curve as constructed, therefore, does not represent the relation of price to quantity purchased with any theoretical accuracy at all, and it represents the psychological value of the satisfaction secured in a fluctuating unit.

We will begin with the latter difficulty. How can we maintain the stability of our psychological unit throughout a series of estimates? What we really want is to fix in our own minds or the mind of our informant the actual psychological magnitude represented by the objective unit at the

margin of our current expenditure; and then to estimate *in that unit* the significance of small increments of the commodity at various margins. We should then have, for any given quantity consumed, what we set out to obtain, viz. an evaluation in a stable unit of the total estimated satisfaction enjoyed, as distinct from the sum paid. These estimates, however, are such as we could only imagine experts trained in a psychological laboratory attempting to make. The naïve, however careful and acute, answers we could expect from practical administrators would never be based on so subtle a conception as that of the psychological unit. We should have to assist our informant by putting our questions in some such form as this: "If when you had bought your tea for the month and paid for it at market prices, you lost half, three-quarters, nine-tenths, or all of your stock, what in each case would you pay for a first small increment, sooner than go without it?" The smallness of the increment estimated would reduce to a vanishing point the reaction of the sum to be paid upon the psychologic value of the money unit, and the fact that in every case the full amount that is actually paid for the commodity, and no more, is already written off, would keep that psychological value uniform. The ingenious reader may still think of disturbing influences, the shock of the loss, the changed significance of other enjoyments caused by the reduction in the supply of tea and so forth; and he may imagine any system of discounts that pleases him. It is clear that in any case absolute fixity of the psychological unit is only an ideal conception, and that actual estimates in money will never be more than approximately consistent in their psychological significance. The essential point is that the total psychic value of the satisfaction derived from the consumption of a given amount of a commodity is a finite quantity, capable of ideal evaluation in a fixed unit, and that over a vast field of our current expenditure it exceeds, in our own estimate, the value of the alternatives we relinquish for it.¹ This total area of satisfaction may, in theory, be represented accurately by a figure which sets forth the marginal significance of every successive increment of the commodity; but if we have taken as our psychic unit the satisfaction which the money unit

¹ But compare the following chapter.

commands at the actual margin of our expenditure under existing conditions, then any hypothesis which sensibly changes those conditions (as by increasing or diminishing the amount actually spent on our commodity) will change the significance of the unit; and therefore, if we measure penny or shilling rates on the axis of Y , it follows that the same figure cannot represent, with theoretical accuracy, the meaning of a number of different hypotheses, regarded as co-existing. Given any price and the actual administration of resources that corresponds to it, we can ideally construct a curve of total satisfaction, the unit of which corresponds to the marginal satisfaction now secured by a penny or a shilling; but if the price changes we cannot preserve the same figure and get an accurate result by simply changing the point on OX at which we erect a perpendicular to cut the curve; for under the changed conditions the satisfaction secured by a penny or a shilling will have changed.

I have been careful to speak of the Figure as giving, ideally, a representation of the total satisfaction derived from the consumption of Ox , in the mixtilinear area above it. I have not said that the surplus of satisfaction over payment would be accurately represented by the area yy_0p . For this again would only be an approximation. In evaluating the price actually paid at Op our Figure implies that if the market for the commodity in question were closed, or if the commodity ceased to exist, the purchaser, while losing the total area above Ox , would gain the released area of the rectangle Op . This means that the whole of the money now spent on this commodity could be expended on other commodities at a marginal significance represented by xp or Oy . But theoretically this is not true, for if the supplies of other commodities were increased, it would of course be at a declining significance, and consequently, when the whole sum Op had been distributed amongst them, their marginal values would have declined to some extent, however small, from the height xp . Some portion of them, therefore, would have less value than if their marginal significance had remained at xp ; and in the sum they will not equal Op . And here again, as we recede from the actual point of departure towards the origin, there will be another source of

Interpreta-
tion of the
rectangle of
payment.

disturbance in the psychic significance of the money unit, independent of the advancing margin, viz. the change in the whole significance of remaining sources of satisfaction as the one to which the Figure refers dries up. Here again, therefore, all attempts to guard against and discount sources of disturbance in the psychic value of our objective unit must be at once subtle and clumsy. The only ideal method is to conceive of a mind trained to hold a psychic magnitude firmly and apply it consistently as a unit. That magnitude would be the satisfaction represented by the money unit under existing conditions, but it would be applied to hypothetically changed conditions directly, and not through the convenient but treacherous intervention of a money unit which might be perpetually changing its significance.

If we traced our original curve with a stable psychic unit, based on the satisfaction secured by a penny or a shilling at present margins, and if we then allowed for the decreasing values of other commodities as the margins advanced, represented by a decline in the height of the ordinates as we pass from xp to Oy , we should have a consistent representation of total satisfaction, and of surplus of satisfaction over the sacrifice represented by the price, corresponding to the actual state of things. It would shew how much satisfaction I get and how much I pay for it, measured in a stable unit. But it would not give us accurate information as to any other than the actual state of things.

If, on the other hand, we were to ask, not "how much would you give for an ounce of tea under such and such circumstances?" but "how much tea would you buy if it were such and such a price?" we should get a curve with just the opposite characteristics. It would give us information about a number of different hypothetical conditions, but its different parts would have no consistent significance. Thus, by asking "how high would the price of tea have to rise before you would stop buying it altogether?" we might find a point on the axis of Y , and then, by asking how much would be bought at the several prices descending from that to zero, we might obtain points on a curve which would accurately represent the relation between price and quantity purchased for every

Curves of
price-and-
quantity-
purchased.

hypothesis at once. But on each hypothesis the psychological significance of the unit would be different, and as it would always make a (theoretical) difference whether the whole sum represented by the mixtilinear area above any abscissa, or only that represented by the rectangle, were paid, the area would never represent accurately either the total sum that the consumer would pay for the amount Ox , or its psychological evaluation in any fixed unit.

A curve, therefore, which professes to give, for every price, (1) the quantity that would be purchased at that price, (2) either the pecuniary or the psychic evaluation of the total satisfaction it would yield, can only be a compromise, for it endeavours to comply with two incompatible sets of conditions. Its construction would illustrate the principle of "temperament" by which a note on the piano which is neither D sharp nor E flat, but a compromise between them, is made to do duty for both alike. This is only possible if the interval between them is small. In our case the errors involved in confounding the two curves become negligible in proportion as the total expenditure on the commodity in question is a negligible part of the man's whole income.

The psychological curve always remains the ultimate and basal fact, and though we can never rely on its precise evaluations it is essential that we should form a precise conception of its nature and should realise that it has a definite value. The price-and-quantity-purchased curve is the most accessible and is the one with which we shall usually work; but unless the contrary is expressly stated we shall assume that our curves have a "temperament" which allows us to read them either way.¹

¹ Cf. Appendix to Chapters II. and III. pages 490-492.

CHAPTER III

ON THE NATURE OF CURVES OF TOTAL SATISFACTION

SUMMARY.—*Curves of total satisfaction are purely abstract ; that is to say, they represent the subjective value attached by a consumer to each increment of the commodity, or the amount he would purchase at any given price, apart from any consideration of the causes that might be supposed in actual experience to limit his supply or raise the price of the commodity, and apart from all reactions upon the price of other commodities. They are also isolated ; that is to say, we cannot conceive of a system of such curves being so constructed as to be valid simultaneously. Nor can we sum their areas, taken successively, without omitting some values and counting others more than once. Nor can we read on them the effect of a rise or fall in the consumer's income. Nevertheless their general form has a high theoretical significance. Communal curves of price-and-quantity-saleable cannot be interpreted psychically, though they rest on a psychic basis. A system of such curves cannot possess simultaneous validity.*

The refinements dwelt upon in the preceding chapter are usually ignored. A curve of price-and-quantity-that-would-be-purchased is supposed to be constructed by a direct process of estimates ; and its area is taken to represent the total satisfaction accruing from the consumption of any given amount of the commodity, while the rectangle of price-multiplied-by-quantity is taken to represent the value of the sacrificed alternatives, the surplus satisfaction being secured without corresponding sacrifice or payment. But, independently of the difficulties thus ignored,

Ideal and
isolated
character of
personal
curves.

the legitimacy of the whole conception has been seriously challenged. Probably this is due to the fact that a personal curve of total utility, though its formation is in itself entirely legitimate, is nevertheless of such an ideal and isolated character, that it cannot be regarded as co-existing with other curves of the like nature, for the same individual, nor can it, and its analogues for other individuals, be made, as they stand, the basis for the calculation, by summation, of a communal curve of the one commodity. And therefore when we try to bring a curve of this nature into relation with any practically realisable hypothesis as to the conditions of markets, it assumes an elusory and evasive character which has tempted the bewildered and impatient student to fling it aside as a mere illusion. All this must now be explained.

We shall best avoid the confusions in which the controversy has often been entangled, and shall at the same time best vindicate the fundamental value and significance of the method itself, by examining more closely the meaning of the condition that "other things must remain unchanged" while we are obtaining our successive data as to how much of the commodity the consumer would purchase at such and such prices. To begin with, amongst the other conditions that are to remain unchanged, we must include the power of purchasing substitutes at the prices now current. For example, when our housekeeper is considering how much tea she would buy if it were 6s. a pound, she will probably think of herself as increasing her purchases of coffee or cocoa as she contracts her purchases of tea; and she will suppose that she will still be able to buy coffee and cocoa at the present prices. Now this shews us at once the isolated nature of our hypothetical tea curve. For suppose we had constructed a coffee curve, as well as a tea curve, on the same principles. We should then find that the conditions we supposed to be stable when we were drawing up our tea curve included the possibility of getting more coffee at the present price; and, in like manner, the conditions we supposed to be stable when we were drawing up our coffee curve will have included the possibility of buying tea, as required, at the present market price. Thus, as soon as we suppose the price of tea to rise, we are violating

Meaning of
the condition
"other things
remaining the
same." Sub-
stitutes.

one of the conditions on which the validity of our coffee curve depends; and, in like manner, if we supposed the price of coffee to change, we should thereby be violating one of the conditions on which the validity of our tea curve depends; for it is sufficiently obvious that the amount of coffee which a housekeeper would buy at any given price might be affected by a change in the price of tea; and *vice versa*. It seems impossible, then, even ideally to draw up a system of curves which shall be valid simultaneously; for any curve purports to represent a number of simultaneous possibilities, indicating what quantities would be purchased at any given price; but a change in the price of any one of the commodities will, or may, affect the quantity of other commodities that would be taken at *any* given price. That is to say, if we change our supposition as to the price of any one commodity, that very supposition will change the form of the curves of other commodities, throughout their course. This perhaps needs some further elaboration and explanation.

Let us start on the assumption that the consumer's income is as a matter of fact distributed in a certain way. He buys *Reaction of changed supply of one commodity on the form of the curves of other commodities.* Oa of commodity A at the price aa , Ob of commodity B at the price $b\beta$, Oc of commodity C at the price $c\gamma$, etc. We construct the curves severally as in Fig. 16, on the principles already illustrated, in every case starting from the same initial hypothesis. Each commodity is measured on the axis of X in its own conventional unit, but the unit on the axis of Y is uniform. We can now suppose *any one* of the curves (say the curve of B) to set forth (as a first approximation, subject to the secondary inaccuracies and inconsistencies dwelt on in the last chapter) the marginal significance of B at any point of supply, the quantity that would be purchased at any given price, and the surplus of satisfaction over enjoyment attendant on the consumption of any quantity, provided always that A, C, etc., can be obtained in any quantities desired at the prices aa , $c\gamma$, etc. But the moment we suppose the price of B to rise and the consumption to contract we may find the consumer taking more of A or C as a substitute, and in that case Oa would no longer represent the amount of A that would be consumed at the price of aa . Nor would the curve as it stands

(unless by accident) represent the relation between price and quantity at any other point either. The curves of A and C therefore may change their form for every value of $b\beta$ and are drawn up on the supposition that it is constant, whereas the curve of B is drawn up expressly to illustrate the significance

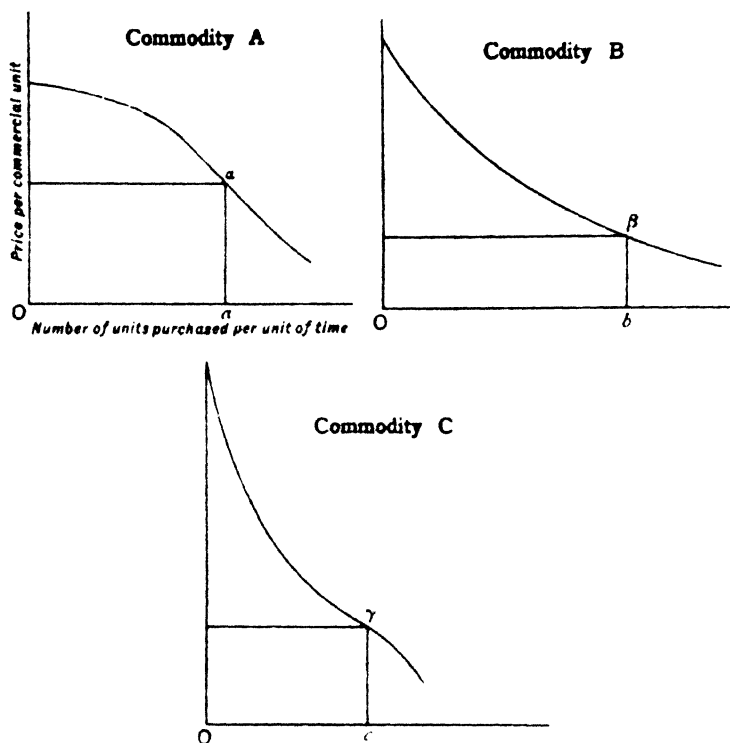


FIG. 16.

of changes in that value, regarded as causes or effects of a change in the magnitude of Ob .

In constructing the curve of B we must be supposed to register the value of $b\beta$ for any value of Ob , or *vice versa*, as the resultants of all the complex readjustments of expenditure caused by a change of supply, or a change of price, in B, the prices of A, C, etc., remaining constant. And if we start in every case from the actual prices aa , $b\beta$, $c\gamma$, etc., we may thus trace the curves of A, B, C, etc., severally and independently, and any one of them will then be valid as long as all the